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## The analysis of dependence of the level of operational costs and production outputs upon geological and mining conditions in selected hard coal mines in Poland

### 1. Introduction and literature review

Hard coal mining is one of the key industries in Poland. At present, the country has 20 active collieries, producing steam and coking coal. Many experts consider such industrial activity as burdened by numerous risks and being capital intensive (Saługa 2009; Dychkovskiy et al. 2018). Hard coal mining has recently been under pressure of falling coal prices on world markets, coupled with the rising costs of services (U.S. EIA 2020). At the same

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time, coal mining conditions have been deteriorating, mining depth and temperature have been increasing, distances for haulage of materials and transport of workforce/crews have been extending, effective time of work has been decreasing, while threats have been intensifying (Kopacz 2017), and the seams mined contain increasing amounts of gangue (ARP 2019). For many years now, restructuring processes have been going on, with the aim of improving the economic efficiency of the entire hard coal mining sector (Szlązak 2004; Sobczyk et al. 2020). The above, along with many other factors make coal mining ever less profitable and strongly dependent upon economic cycles (Fałtyn and Naczyński 2018; Baran et al. 2018).

The possibilities of improving that situation may be sought in better planning processes. The improvement would have to aim at such planning of production which would be as predictable as possible, being on the other hand, economically efficient. In that respect, it appears instrumental to plan the future mining of longwalls being fully aware of the geological and mining conditions in which the mining will take place, as well as the resulting economic consequences, in the form of expected longwall cost over the entire lifecycle, or the assumptions concerning expected output level.

This publication belongs to the line of research conducted by the authors, documented, among others, in (Sobczyk 2008; Sobczyk and Kopacz 2018). It is an attempt to identify the relationship between some geological and mining parameters and longwall cost levels, or the theoretical coal net output defined for them. This attempt is also a verification of what has been contemplated so far concerning the existence of relations between the level of nuisance and the level of operating costs incurred at the phase of fitting, mining, and salvage operations. It has to be stressed here that previous research of the authors was based mainly on the analysis of those relations on the basis of correlations and simple regression models. In this publication, we have introduced aspects of statistical modeling, in order to determine acceptable models and forecasts of longwall operating costs, and the theoretical production capacity of longwalls. It is worth stressing that the results of such research may be put to practical use – in the course of planning future longwall mining in the analyzed coal mines.

## 2. Research objective, methodology and data

### 2.1. Research objectives

The paper presents an attempt to verify the hypothesis about the existence of the relationships between the level of nuisance of geological and mining parameters and the production costs in longwalls and the level of coal net output.

The main aim of the work was to develop a statistical model representing the relationships between longwall operating costs, coal output and geological and mining factors respectively.

In the research process, two statistical models were built for the following dependent variables: unit operational cost (Model 1) and coal net output (Model 2). The definitions for all of them are given in section 2.4. The models also cover: interpretation of the impact of the independent variables on the dependent variables and point predictions.

The first part of the work was devoted to establishing the relationships between longwall operational costs, their production outputs and geological and mining conditions (factors) represented by the indicator of technical nuisance (WUe, WUt). The aggregated impact of natural hazards, coal seams specification and longwall technical parameters – so called technical nuisance – was determined with use of an Analytic Hierarchy Process (AHP), more precisely described in the following publications (Sobczyk 2008; Sobczyk and Kopacz 2018). Statistical models were built on the past production performance of five Polish coal mines, than empirically tested. The impact of individual parameters on the longwall operations was determined on the basis of variability of geological and mining conditions in 120 longwalls in the 2010–2019. Finally, identified relationships allowed to formulate the numerical prediction of the unit production cost and longwall coal net output with reference to the level of expected technical nuisance to be formulated. The prediction period was established for the years 2020–2030. Based on that, we were able to formulate our opinion on the expected unit production cost and coal net output of 230 longwalls planned for exploitation in those mines.

The scheme of the research process is shown in Figure 1.

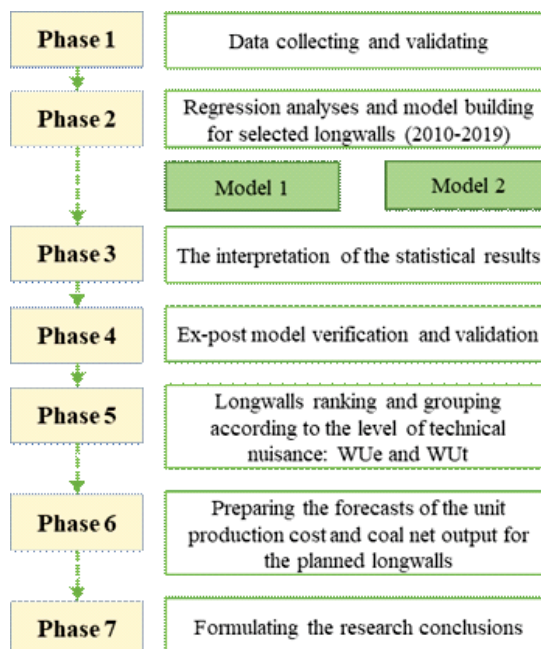


Fig. 1. The research process. Source: own study

Rys. 1. Etapy procesu badawczego

## 2.2. Geological and mining conditions which influence mining in hard coal mines

Mining conditions in underground hard coal mines are determined by natural, technical, and organizational factors, which jointly determine the possibilities and ways of mining extraction. They are decisive for safety, mining efficiency and production results of coal mines. By taking significant factors in the analysis into account, those which influence the mining process itself, it is possible to quantify risks related to mining, namely those which stem from geological and mining conditions, and to demonstrate their influence upon the mining cost level, as well as mining limitations, thus to focus on longwall mining itself (Magda 2016).

Geological and mining conditions comprise all the factors related to the occurrence, development, and mining of hard coal in the deposit. These factors result from the geology and structure of the deposit, as well as hydro-geological, gas-related, geothermal, and geo-technical conditions. Some of those conditions may change with the progress of mining in the colliery, whereas others may alter due to the influence adjacent mines/collieries.

Applying the hierarchical approach proposed by Sobczyk (Sobczyk and Kopacz 2018), geological and mining conditions may be divided into four groups of factors which influence the general level of mining nuisance, namely:

- ◆ natural hazards,
- ◆ coal seam parameters,
- ◆ mining (technical) parameters,
- ◆ environmental factors.

Natural factors and seam parameters are connected with the geology and structure of hard coal deposits. In particular, they comprise such factors as: seam thickness, partings in the seam, lithologic structures in the roof and floor, the occurrence of faults and other geological disturbances, coal seam dip, cracks in coal seams and surrounding rocks, the occurrence of thinning and wash-out erosion in seams, petrographic composition of coal (Paździora 1988). The factors considered to be natural hazards and deposit (seam) parameters also comprise hydro-geological, gas, thermal, and geotechnical conditions. Hydro-geological conditions are determined by the site hydrography, geological structure, type of overburden, and past mining activities. Gas conditions of the deposit are connected with the presence or absence of methane (the amount of methane present matters substantially). Thermal conditions of the deposit depend on the rock temperature. This, in turn, is connected with the geothermal degree in the deposit. Geotechnical conditions are linked with the susceptibility of coal and surrounding rock to bumping, roof and floor classes next to the coal seams, as well as workability of coal and gangue. From the above factors, the following have been selected for further statistical analyses: natural hazards, tectonics of the analyzed deposit, sedimentation conditions, depth at which the seam occurs, or dirty bands thickness. Mining practice indicates that mining activity in the presence of natural hazards – due also to legal, organizational, and health & safety limitations – is connected with substantial

spending on preventive activities, which aim at creating safe work conditions. Natural hazards (e.g. methane, bumps) limit longwall advance and coal output.

Technical (mining) parameters are all the parameters related to the physical location of workings (longwalls), geometric parameters of those longwalls, as well as the factors defined as interaction between the longwall and rock mass. In the case of technical parameters, the analyses took into account such criteria which, in accordance with the knowledge the authors accumulated and with mining experience in collieries, significantly influence mining costs and longwall production results. They comprised, in particular: location of longwalls within the mining area, which is connected with the logistics of haulage and transport, geometrical parameters of longwalls which – together with seam parameters – determine the coal output from a given longwall which, taking the relatively high share of fixed costs into consideration, translates into the level of unit costs.

It should be pointed out that apart from the factors influencing geological and mining conditions of production, the features which also characterize the influence of mining upon the environment have been taken into account in the statistical analysis. The waste rock amount is included among the environmental factors, as it increases the total run-of-mine as well as production cost.

### 2.3. Data analysis methods

The multiple linear regression model used in the study can be expressed in general as

$$\ln y = \mathbf{X}\boldsymbol{\beta} + \varepsilon \quad (1)$$

where  $y = (y_1, y_2, \dots, y_n)'$  is a  $n \cdot 1$  vector of observations of the dependent variable.  $\mathbf{X}$  is the  $n \cdot (k + 1)$  matrix of observations for each of the explanatory variables (first column is a unit constant),  $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2, \dots, \beta_k)'$  is  $(k + 1) \cdot 1$  vector of  $k$  regression coefficients and intercept and  $\varepsilon = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)'$  is a  $n \cdot 1$  vector of random error components. Regression coefficient are estimated using the Ordinary Least Squares (OLS). In model (1), the  $j$ -th regression coefficient  $\beta_j$  approximately represents the expected change in by  $100 \cdot \beta_j$  percent per unit increase in  $j$ -th independent variable  $X_j$ , assuming the unchanged levels of all the remaining independent variables. Each independent variable ( $X_j$ ) was standardized by standard deviation.

The following assumptions underlying OLS regression are especially important to verify before proceeding to the interpretation of the model outcomes:

- ◆ linearity of the relationship between the predictors and the outcome variable,
- ◆ minimal multicollinearity among the independent variables,
- ◆ normality of error term,
- ◆ homoscedasticity and independence of errors,
- ◆ proper model specification (e.g. inclusion of all relevant variables and exclusion of irrelevant variables).

Without making sure that model assumptions are tenable, the results of estimation, point and interval prediction may be misleading.

At the beginning, the appropriate functional form for all variables was revealed using graphical methods, to assure the proper statistical modelling. All appropriate pairwise scatterplots were investigated to search for non-linearity or any atypical data patterns e.g. outlying observations.

One of the main assumptions for the OLS regression is the homogeneity of variance of the residuals. The literature includes many graphical and non-graphical methods for detecting heteroscedasticity. A commonly used graphical method is to plot the residuals versus fitted (predicted) values, while the Breusch–Pagan test (Breusch and Pagan 1979) is a popular non-graphical method for detecting heteroscedasticity. The null hypothesis of the Breusch–Pagan test claims that the variance of the residuals is homogenous. Under the null hypothesis of homoscedasticity the test statistic is asymptotically distributed as  $\chi^2$  with  $k$  degrees of freedom (Wooldridge 2013).

Normality of residuals is required in small samples for valid confidence interval estimation and hypothesis testing. Normality is not required to obtain unbiased estimates of the regression coefficients. OLS regression merely requires that the residuals (errors) be identically and independently distributed. Violations of normality assumption create problems for determining whether model coefficients are significantly different from zero and for calculating confidence intervals. Also if the error distribution is strongly non-normal, prediction intervals may be too wide or too narrow, depending on the tail behavior of the error distribution. There are several methods of assessing whether error distribution is normal. They fall into two broad categories: graphical and statistical. The common graphical techniques are Q-Q probability plots and cumulative frequency (P-P) plots. Many statistical tests are used to assessing normality e.g. Shapiro–Wilk (SW), Kolmogorov–Smirnov (KS), Lilliefors, Anderson–Darling (AD), Jarque–Bera test. Monte Carlo simulation studies have found that Shapiro–Wilk test has the best power for a given significance level, followed closely by Anderson–Darling when comparing SW, KS and AD tests (Razali and Wah 2011). SW test was designed to test for normality for *small* data-size ( $n < 50$ ), but is also recommended for larger samples. The SW tests the null hypothesis that a sample  $x_1, \dots, x_n$  came from a normally distributed population. The test statistic is

$$W = \frac{\left( \sum_{i=1}^n a_i x_{(i)} \right)^2}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

where  $x_{(i)}$  is the  $i$ -th smallest value in the sample. The coefficients  $a_i$  are given by  $(a_1, \dots, a_n) = (m'V^{-1})/C$ , where  $C$  is a vector norm  $C = \|V^{-1}m\|$  and the vector  $m = (m_1, \dots, m_n)'$  is made of the expected values of the order statistics of independent and identically distributed random variables sampled from the standard normal distribution and is the covariance matrix of those normal order statistics (Shapiro and Wilk 1965). Some minor modifications to the

SW test have been suggested by Shapiro and Francia (Shapiro and Francia 1972), Weisberg and Bingham (Weisberg and Bingham 1975) and Royston (Royston 1982).

Multicollinearity occurs when two or more predictors in the model are correlated and may provide redundant information about the response variable. In this study the Variance Inflation Factor (VIF) is used to check for multicollinearity. VIF measures how much the variance of the regression coefficients is inflated by multicollinearity. If VIF equals zero, there is no correlation between the independent variables. Various recommendations for acceptable levels of VIF have been published in the literature. Perhaps most commonly, a value of 10 has been recommended as the maximum level of VIF (Hair et al. 1995; Kennedy 1992; Marquardt 1970; Neter et al. 1989). However, a recommended maximum VIF value of 5 (e.g., Rogerson 2001) and even 4 (e.g., Pan and Jackson 2008) can also be found in the literature. The reasonable rule of thumb is that VIFs exceeding 5 warrant further investigation, while VIFs exceeding 10 are a sign of serious multicollinearity requiring intervention.

A model specification error can occur when one or more relevant variables are omitted from the model or one or more irrelevant variables are included in the model. If relevant variables are omitted from the model, the common variance they share with included variables may be wrongly attributed to those variables and the error term is inflated. On the other hand, if irrelevant variables are included in the model, the common variance they share with included variables may be wrongly attributed to them. Model specification errors can substantially affect estimates of regression coefficients. One of the regression specification tests is the Ramsey Regression Equation Specification Error Test (RESET) for omitted variables. It creates new variables based on the predictors and refits the model using those new variables to check if any of them are significant. The null hypothesis of the RESET test claims that the model has no omitted variables. The RESET tests whether non-linear combinations of the fitted values help explain the response variable. The intuition behind the test is that if non-linear combinations of the explanatory variables have any power in explaining the response variable, the model is misspecified in the sense that the data generating process might be better approximated by a polynomial or some other non-linear functional form.

## 2.4. Data

To determine the impact of geological and mining conditions on the operation of individual longwalls and their costs, the statistical models were elaborated with use of a set of criteria presented in Table 1. Input data was collected and grouped from: natural hazards, coal seam parameters, mining (technical) parameters and environmental criteria.

To do so, a two step procedure was elaborated. The first step, based on expert judgment, was to indicate explanatory variables of strategic importance for the forecast of the longwall's costs and coal net outputs. The second step was to eliminate explanatory variables

Table 1. Codenames and definitions for the dependent and independent variables

Tabela 1. Kody i definicje zmiennych zależnych i niezależnych

Codename	Definition
$Y_1$	Total longwall unit cost
$Y_2$	Longwall coal net output
$X_1$	Methane hazards (class)
$X_2$	Susceptibility of coal to self-ignition (group)
$X_3$	Gases and rocks outbursts (category)
$X_4$	Rock bumps in the seam (grades)
$X_5$	Climatic hazard (primary rock mass temperature °C)
$X_6$	Tectonics (fault occurrence indicator m/m <sup>2</sup> of the longwall surface)
$X_7$	Sedimentation disorders (scale 1 to 5 (1 – none, 5 – very large))
$X_8$	Depth of the seam (m)
$X_9$	Partings in the seam (cm)
$X_{10}$	Workability (class)
$X_{11}$	Floor conditions (class)
$X_{12}$	Roof conditions (class)
$X_{13}$	Haulage and transport distance (m)
$X_{14}$	Arrival/transport time of the crew to the longwall (minutes)
$X_{15}$	Distance from the intake shaft (m)
$X_{16}$	Longwall length (m)
$X_{17}$	Longwall panel length (m)
$X_{18}$	Longwall height (m)
$X_{19}$	Occurrence of exploitation events in the longwall surroundings (scale 1 to 5 (1 – very small, 5 – very large))
$X_{20}$	Amount of waste rock in the ROM coal (%)

Source: own study.

( $X_i$ ) that were strongly correlated and to determine the representatives for a given nuisance group. Finally, 20 variables (Table 1) were included in the range of input data to regression modelling.



### 3. Results

Table 2 contains descriptive statistics and VIF for all independent variables  $X_1, X_2, \dots, X_{20}$  described in Table 1. Table 3 contains the results of the estimation of two multiple regression models for dependent variables  $Y_1, Y_2$  described in Table 1. The statistical assumptions of two regression models (obtained from a backward step-wise procedure) have been checked and the results are presented in Table 2 and 3. First, the Variance Inflation Factor (VIF) was used to check for the multicollinearity of all predictors and none of the VIF values have exceeded the acceptable level of VIF equal to 5. Afterwards, for all considered models

Table 2. Descriptive statistics and VIF

Tabela 2. Statystyki opisowe i VIF (test współliniowości)

Predictor	Mean	Standard deviation	VIF
$X_1$	0.451	0.161	2.85
$X_2$	0.110	0.073	1.89
$X_3$	0.127	0.280	1.92
$X_4$	0.125	0.171	1.81
$X_5$	39.367	4.572	4.70
$X_6$	0.001	0.002	1.22
$X_7$	1.808	1.027	1.70
$X_8$	867.967	114.359	4.25
$X_9$	0.212	0.240	1.82
$X_{10}$	0.113	0.100	2.78
$X_{11}$	0.670	0.267	2.44
$X_{12}$	0.748	0.383	2.29
$X_{13}$	3682.225	1 930.692	3.16
$X_{14}$	46.716	18.722	2.34
$X_{15}$	2704.500	915.386	1.79
$X_{16}$	198.582	45.761	1.78
$X_{17}$	779.693	399.052	1.47
$X_{18}$	2.511	0.732	2.27
$X_{19}$	2.342	1.076	1.53
$X_{20}$	0.264	0.114	2.40

Source: own study.

Table 3. Multiple linear regression results

Tabela 3. Wyniki analizy regresji

Predictors	Predicted variable	
	Model 1 ln $Y_1$	Model 2 ln $Y_2$
C3	-0.421*** (0.088)	-0.406*** (0.095)
C5	–	-0.207* (0.126)
$X_1$	0.120*** (0.029)	-0.043 <sup>(e)</sup> (0.035)
$X_3$	0.094*** (0.029)	-0.086*** (0.029)
$X_4$	0.032 <sup>(e)</sup> (0.028)	-0.039 <sup>(e)</sup> (0.029)
$X_6$	0.106*** (0.028)	-0.096*** (0.028)
$X_9$	–	-0.121*** (0.028)
$X_{15}$	0.161*** (0.029)	–
$X_{16}$	–	0.079** (0.033)
$X_{17}$	-0.217*** (0.029)	0.072** (0.030)
$X_{18}$	–	0.184*** (0.031)
C	3.881*** (0.132)	6.978*** (0.225)
$R^2$	0.5925	0.5307
Adjusted $R^2$	0.5671	0.4876
RMSE	0.297	0.292
B–P test	0.08 p = 0.7718	0.13 p = 0.7214
Shapiro–Wilk test	0.99239 p = 0.75701	0.99232 p = 0.75064
RESET test	1.89 p = 0.1352	3.10** p = 0.0298

Notes: \*\*\* denotes statistical significance of regression coefficient at 0.01, \*\* at 0.05, \* at 0.10 and <sup>(e)</sup> means that variable remains in model because of substantive reasons. In parentheses are standard errors. C is an intercept. RMSE is the estimated standard error of the regression. B–P test denotes Breusch–Pagan and Shapiro–Wilk test is used to test residual normality. RESET test denotes the Ramsey RESET test.  $C_{1-5}$  denotes coal mines in the models.

Source: own study.

homoscedasticity and normality of residuals were verified by means of the Breusch–Pagan and Shapiro–Wilk tests. In all cases, the Breusch–Pagan test failed to reject the hypothesis about the homoscedasticity of residuals. The Shapiro–Wilk test did not reject normality of residuals at a 0.05 significance level for both models.

The coefficients of determination ( $R^2$ ) for models 1 and 2 were 0.5923 and 0.5307. This value shows that a large portion of each dependent variable variability is unaccounted for by the models. However, this behaviour was to be expected as in practice complex economical and industrial processes are difficult to measure reliably.  $R^2$  on a lower side also indicates that the dependent variable may be affected by a host of other factors in addition to the ones considered in the analysis (Moksony 1990). Large uncertainty causes the prediction intervals to be wide but point predictions and model coefficient interpretations may still offer a valuable guidance for budgeting and economic decision-making in an information-deprived environment.

In the next step, regression coefficients presented in Table 3 are technically interpreted. In order to avoid unnecessary repetitions in all subsequent interpretations of regression coefficients it is implicitly assumed that all other predictors remain at their constant levels and also that all predictors impacts are on average.

The results of the studies are regression models, which enable to estimate total unit costs as well as coal net output for longwalls, on the basis of factors resulting from natural hazards, geological structure of the deposit (seam), as well as technical factors. Structural parameters of the models characterize the strength and direction of influence of specific independent variables  $X_i$  upon dependent variables  $Y_i$ . The influence of particular variables upon dependent variables  $Y_1$  and  $Y_2$  is illustrated by Figure 2 and 3.

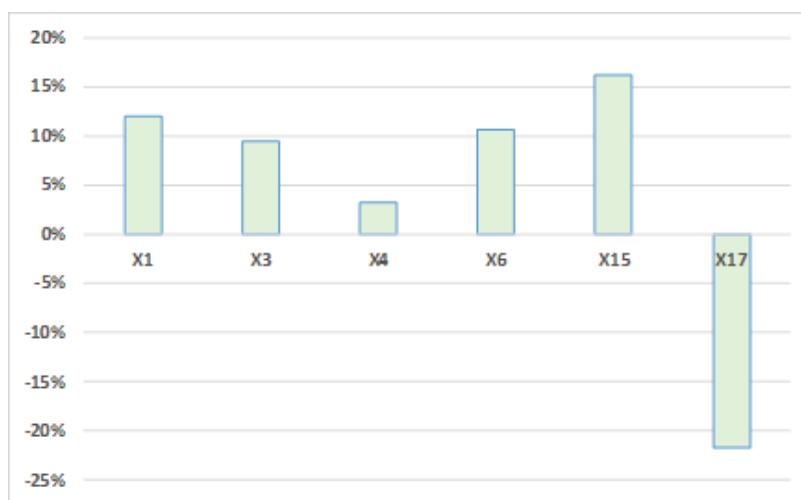


Fig. 2. Relative impact of the  $X_i$  parameters on  $Y_1$  (Model 1) – total unit costs  
Source: own study

Rys. 2. Wpływ zmiennych objaśniających  $X_i$  na zmienne objaśniane  $Y_1$  (Model 1) – koszty jednostkowe całkowite ścian

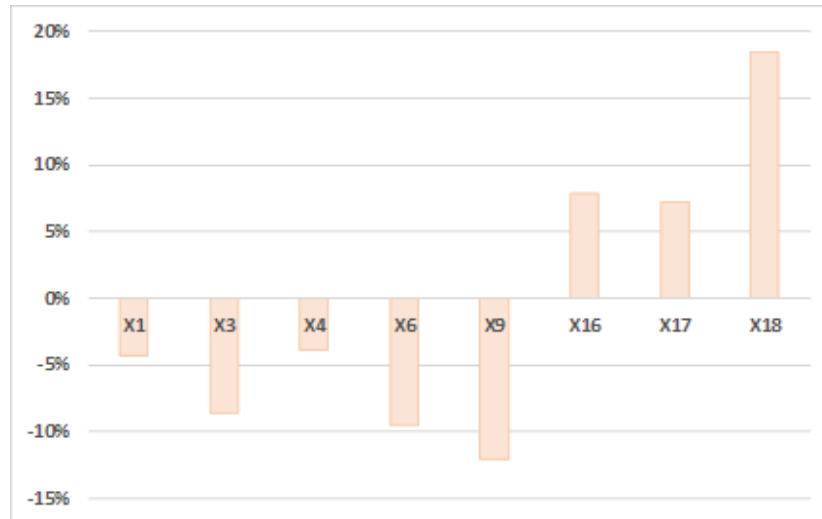


Fig. 3. Relative impact of the  $X_i$  parameters on  $Y_2$  (Model 2) – coal net output for longwalls  
Source: own study

Rys. 3. Wpływ zmiennych objaśniających  $X_i$  na zmienne objaśniane  $Y_2$  (Model 2) – zdolność produkcyjna (netto) ścian

An increase of  $X_1$  (methane hazard) by one standard deviation causes an increase of  $Y_1$  by 12% and a decrease of  $Y_2$  by 4.3%. The predictor  $X_3$  (gases and rocks outbursts hazard) has a statistically significant impact on both  $Y_1$  and  $Y_2$ . An increase of  $X_3$  by one standard deviation causes an increase of  $Y_1$  by 9.4%, and a decrease of  $Y_2$  by 8.6%. An increase of  $X_4$  (rock bumps in the seam) by one standard deviation causes an increase of  $Y_1$  by 3.2%, and a decrease of  $Y_2$  by 3.9%. The predictor  $X_6$  (fault occurrence indicator) has a statistically significant impact on  $Y_1$  and  $Y_2$ . An increase of  $X_6$  by one standard deviation causes an increase of  $Y_1$  by 10.6%, and a decrease of  $Y_2$  by 9.6%. The predictor  $X_9$  (coal partings in the seam) has a statistically significant impact on  $Y_2$  and an increase of  $X_9$  by one standard deviation causes a decrease of  $Y_2$  by 12.1%. The predictor  $X_{15}$  (distance from the intake shaft) has a statistically significant impact on  $Y_1$  and an increase of  $X_{15}$  by one standard deviation causes an increase of  $Y_1$  by 16.1%. The predictor  $X_{16}$  (longwall length) has a statistically significant impact on  $Y_2$  and an increase of  $X_{16}$  by one standard deviation causes a decrease of  $Y_2$  by 7.9%. The predictor  $X_{17}$  (longwall panel length) has a statistically significant impact on both  $Y_1$  and  $Y_2$ . An increase of  $X_{17}$  by one standard deviation causes a decrease of  $Y_1$  by 21.7%, and an increase of  $Y_2$  by 7.2%. The predictor  $X_{18}$  (longwall height) has a statistically significant impact on  $Y_2$  and an increase of  $X_{18}$  by one standard deviation causes an increase of  $Y_2$  by 18.4%.

## 4. Discussion and conclusions

### 4.1. Ex-post models verification

In order to verify the predictive quality of the models, the predicted values of total unit costs and theoretical production net output were calculated for 120 longwalls, used for the construction of Models 1 and 2. Relative error, calculated as the difference of the empirical and theoretical value related to the empirical value was elaborated for both models. The distributions of relative error for both models are shown in Figures 4 and 5. The basic statistics of Maximum Extreme Value distributions are presented in Table 4.

Analysing the distribution of the relative errors so determined, it can be concluded that:

- ◆ the arithmetic mean of the relative error for model 1 is 5.5%, and for model 2, 4.3% respectively,
- ◆ 90% of all observations are in the range of (–54%; 95%) in model 1 and (–53%; 90%) in model 2,

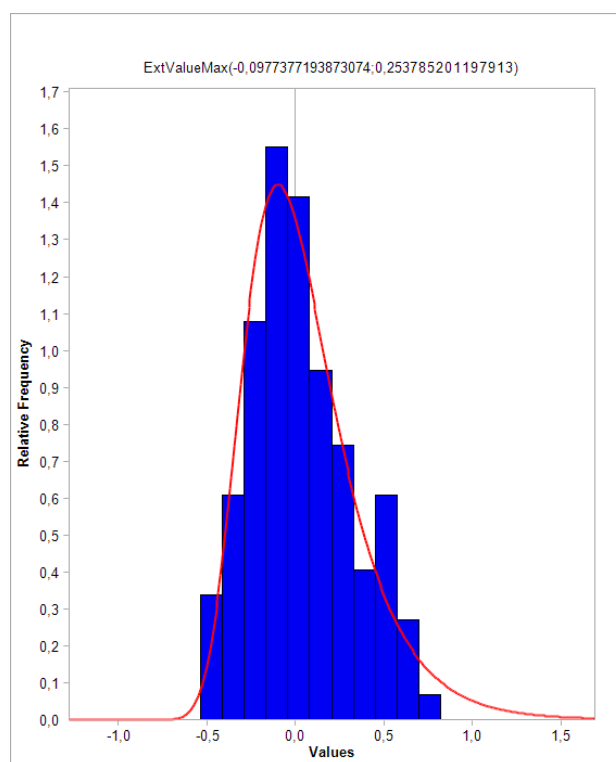


Fig. 4. Distribution of the relative error (Model 1) – ex-post verification  
Source: own study

Rys. 4. Rozkład błędu względnego (Model 1) – weryfikacja ex-post

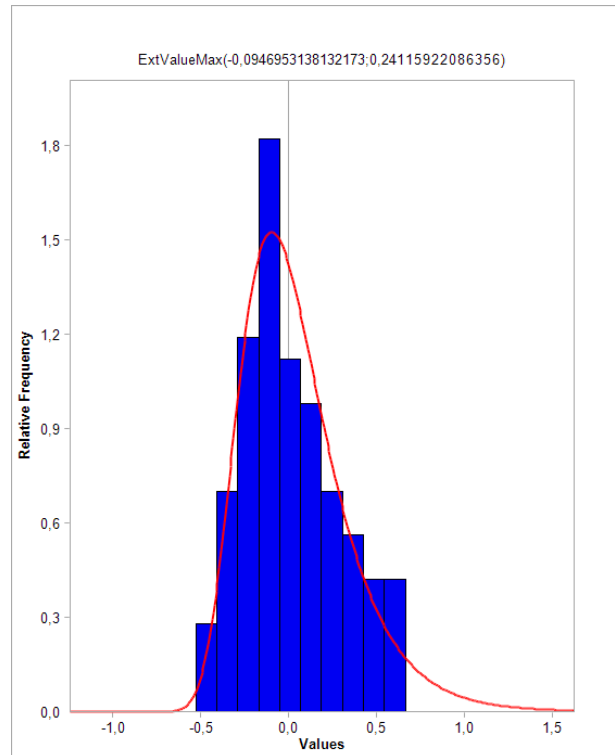


Fig. 5. Distribution of the relative error (Model 2) – ex-post verification  
Source: own study

Rys. 5. Rozkład błędu względnego (Model 2) – weryfikacja ex-post

- ◆ for 90 walls (75% of all observations), the relative error is in the range of  $(-25\%; 37\%)$ ,
- ◆ both distributions are right-skewed, and the best fitted theoretical distribution is the distribution of maximum extreme values.

On this basis, it was considered that the theoretical models (Model 1 and 2), determined on the basis of multiple regression, can be used to make a forecast of unit cost and theoretical coal net output for longwalls planned in the future, taking the technical nuisance of the designed longwall faces into account.

#### 4.2. Forecast of costs of operation and longwall production capacity

The development of regression models – Model 1 (Y1) and Model 2 (Y2) aimed at enabling the estimation of approximate unit costs, as well as forecasted coal net output in the designed longwalls, in the context of differentiated geological and mining conditions.

In this specific case, the analysis comprised 220 longwalls in 5 hard coal mines, planned to be mined in the years 2020–2030.

Table 4. Basic statistic of the relative error distributions – ex-post verification

Tabela 4. Podstawowa statystyka rozkładów błędów względnych – weryfikacja ex-post

Variable Name	Error distribution Model 1 (Y1)	Error distribution Model 2 (Y2)
Location statistics		
Mean	5.2%	4.3%
Minimum	–53.6%	–52.5%
Maximum	94.6%	90.2%
Spread statistics		
St. dev.	0.32	0.31
Variance	0.11	0.09
CofV	6.25	7.22
Shape statistics		
Skewness	1.10	1.23
Kurtosis	5.18	5.89
Percentiles		
5%	–38%	–36%
15%	–25%	–25%
85%	37%	33%
95%	65%	61%

Source: own study.

In order to determine the level of geological and mining conditions nuisance in the analyzed hard coal mines, a pattern of procedures has been used in which the mathematical multiple criteria decision-making process has been applied, abbreviated as AHP (Analytic Hierarchy Process). Using the AHP analysis, [two] nuisance models for geological and mining conditions in the exploitation process have been developed for specific longwalls in hard coal mines: the first one – in the context of production costs, the second one – considering the concentration of extraction from longwalls. AHP models, comprising natural hazards, deposit (seam) parameters, mining (technical) parameters, and environmental factors, served for calculation of nuisance factors WUe and WUt. Those factors determine, in a synthetic manner, the level of nuisance for geological and mining conditions of extraction in the years 2020–2030, in reference to longwall:

- ◆ unit operating cost – WUe, and,
- ◆ coal net output – WUt.

The detailed methodology for assessing the nuisance of geological and mining conditions for exploitation has been presented in the papers of (Sobczyk 2008; Sobczyk and Kopacz 2018).

The result of the procedure was a linear arrangement of exploitation longwalls in accordance with increasing nuisance of geological and mining conditions (determination of longwall ranking). For WUe, a 3-level nuisance scale was introduced (low nuisance: WUe <18, medium nuisance: WUe in the range of <18;23>, high nuisance: WUe >23), whereas for WUt factor a 4-level scale was implemented (low nuisance: WUt <19, average nuisance: WUt in the range of <19;24>, high nuisance: WUt in the range of <24;32>, very high nuisance: WUt >32).

One can expect that the higher the value of nuisance factors attributed to longwalls, the more prominent the influence of geological and mining conditions of exploitation upon the economic results of mining process and production capacity.

Subsequently, in each analyzed coal mine the longwalls to be mined in the years 2020–2030 have been assigned to nuisance groups defined as above. For each nuisance group, the values of all geological and mining criteria that influence the mining process have been averaged. In the next step, using regression models Y1 and Y2, for specific nuisance groups, forecasts of unit costs of production and coal net output have been made. The average forecasted total unit costs for longwalls according to nuisance groups in selected coal mines are presented in Table 5, whereas the average expected longwall coal net output – in Table 6.

In all the coal mines analyzed, the lowest total unit cost occur for longwalls having the lowest nuisance factors, WUe <18 (costs increase with increasing nuisance). In the group of highest nuisance (WUe >23), the average increase of average unit cost by as much as 26% can be noted. The highest relative cost increase occurs in Mine 1, Mine 2, and Mine 5. The average unit cost in Mine 1 increases from PLN 62.5/Mg in longwalls with nuisance factor WUe <23 to PLN 78.5/Mg in the longwalls with the highest values of WUe. On the other hand, in Mine 4, with nuisance factor WUe <18 the average unit cost was PLN 53.9/Mg, increasing to PLN 63.2 /Mg in case of nuisance factor WUe >23. In Mine 5, in turn, the average unit cost increases from PLN 86.5/Mg to PLN 106.7/Mg for the highest nuisance group (increase by 23%). The highest average unit cost of production has been determined for Mine 2, though (in the range of PLN 87.9–109.9/Mg; with average unit cost for longwalls mined in the years 2010–2019 amounting to more than PLN 120/Mg).

In the case of the forecasted coal net output, the highest efficiency occurs in the group of longwalls having the lowest nuisance values, with WUt <19. The average forecasted daily coal outputs decrease with uprising nuisance values. For example, in Mine 4, in the group with highest nuisance (WUt >32) the average coal net output drop of 37% is noted. In that mine, the forecasted daily coal net output is reduced from 3,859 Mg/d for longwalls with lowest nuisance factors, to 2,446 Mg/d in longwalls where nuisance factor WUt values are the highest. On the basis of Table 6, a general conclusion can be made that when the nuisance



Table 5. Average forecasted costs for longwalls according to nuisance groups, in selected coal mines

Tabela 5. Średnie prognozowane koszty ścian według grup uciążliwości w wybranych kopalniach węgla kamiennego

WUe	Mine 1		Mine 2		Mine 3		Mine 4		Mine 5	
	Expected unit operating cost/nuisance group									
	UC	RV	UC	RV	UC	RV	UC	RV	UC	RV
<18	–	–	87.9		83.1	–	53.9	–	86.5	–
18–23	62.5	–	93.1	6%	94.1	13%	50.1	–7%	92.9	7%
>23	78.5	26%	109.9	25%	–	–	63.2	17%	106.7	23%
Average unit cost in the mines (zł/Mg)										
UC	78.2		120.3		102.9		52.0		85.1	

UC – unit operating cost for longwalls, PLN/Mg,

RV – relative value (the value in higher nuisance group/ divided by the value in the lower nuisance group, %).

Source: own study.

Table 6. Average forecasted coal net output for longwalls according to nuisance groups, in selected coal mines

Tabela 6. Średnie prognozowane zdolności produkcyjne ścian wg grup uciążliwości w poszczególnych kopalniach

WUt	Mine 1		Mine 2		Mine 3		Mine 4		Mine 5	
	Expected coal net output/nuisance group									
	P	RV	P	RV	P	RV	P	RV	P	RV
<19	2 431	–	2 295	–	2 331	–	3 859	–	2 298	–
19–24	2 426	0%	2 371	3%	2 483	7%	4 032	4%	2 109	–8%
24–32	2 351	–3%	1 917	–16%	2 215	–5%	3 140	–19%	1 962	–15%
>32	1 846	–24%	1 702	–26%	2 089	–10%	2 446	–37%	1 986	–14%
Average coal output in the mines										
	2 845		1 891		2 615		3 608		2 186	

P – coal net output, Mg/d,

RV – relative value (the value in higher nuisance group/ divided by the value in the lower nuisance group, %).

Source: own study.

factor increases above the value of 32, this causes a drop in productivity by a few dozen percent. However, this develops differently in the coal mines analyzed, which calls for considering each case individually.

## Conclusion

The studies reported in this paper confirm the existence of a relationship between geological and mining conditions on the one hand, and production costs as well as coal net output on the other hand. Thus, the research thesis has been positively verified. Increased nuisance translates, in a non-linear manner, into increases in production costs in the analyzed longwalls, as well as worse/poorer production results. Those results are in line with the intuition and expectations of the authors. The application of regression models provided the possibility of linking geological and mining parameters to specific mathematical functions, which allow to make forecasts concerning operating costs for longwalls and theoretical production capacities of the latter. The models which have been developed are characterized by acceptable quality regarding statistics. The error rate for single ex-post observations does not exceed 5% for each model, while for 75% of observations it is in the range of (–25%; +37%). It should be stressed that the obtained results have a significant practical application, and should be utilized in designing mining operations in hard coal mines, indicating the general direction (trend) of changes in cost levels and output as a function of nuisance concerning geological and mining conditions. Taking such information into account may result in a more conscious designing of mining extraction, bearing the expected production results, quality of gotten coal, and costs in mind.

*The publication was carried out as part of the statutory activity of the Mineral Energy and Economy Research Institute of the Polish Academy of Sciences. The authors acknowledge support from research funds granted to the Cracow University of Economics, within the framework of the subsidy for the maintenance of research potential.*

## REFERENCES

- ARP 2019 – Agencja Rozwoju Przemysłu. Raport o stanie górnictwa węgla kamiennego, Katowice.
- Baran et al. 2018 – Baran, J., Lewandowski, P., Szpor, A. and Witajewski-Baltvilks, J. 2018. *Coal transition in Poland, Options for a fair and feasible transition for the Polish coal sector; IBS*. [Online] [https://coaltransitions.files.wordpress.com/2018/09/coaltransitions\\_finalreport\\_poland\\_2018.pdf](https://coaltransitions.files.wordpress.com/2018/09/coaltransitions_finalreport_poland_2018.pdf) [Accessed: 2020-06-09].
- Breusch, T.S. and Pagan, A.R. 1979. A Simple Test for Heteroskedasticity and Random Coefficient Variation. *Econometrica* 47(5), pp. 1287–1294.
- Dychkovskiy et al. 2018 – Dychkovskiy, R., Falshtynskiy, V., Ruskykh, V., Cabana, E. and Kosobokov, O. 2018. A modern vision of simulation modelling in mining and near mining activity. *E3S Web of Conferences* (60). [Online] <https://doi.org/10.1051/e3sconf/20186000014>.
- Fałtyń, M. and Naczyński, D. 2018. The factors shaping the demand, supply, and prices on the hard coal market. Modelling of possible changes in the long-term horizon. *Polityka Energetyczna – Energy Policy Journal* 21(3), pp. 47–68.
- Hair et al. 1995 – Hair, J.F.Jr., Anderson, R.E., Tatham, R.L. and Black, W.C. 1995. *Multivariate Data Analysis* (3rd ed), New York, US.
- Kennedy, P. 1992. *A Guide to Econometrics*. Oxford: Blackwell.
- Kopacz, M. 2017. The impact of selected geological and mining parameters on the economic evaluation of projects in the hard coal mining industry (*Wpływ wybranych parametrów geologiczno-górnictwowych na ocenę ekonomiczną*).

- na projektów w górnictwie węgla kamiennego). *Studia, Rozprawy, Monografie* 201, Kraków: MEERI PAS (in Polish).
- Magda, R. 2016. Ways of rationalization of unit cost of production in the mining (*Kierunki racjonalizacji jednostkowego kosztu produkcji w przedsiębiorstwie górniczym*). *Inżynieria Mineralna* 17(2), pp. 145–152 (in Polish).
- Marquardt, D.W. 1970. Generalized inverses, ridge regression, biased linear estimation, and nonlinear estimation. *Technometrics* 12, pp. 591–256.
- Neter et al. 1989 – Neter, J., Wasserman, W. and Kutner, M.H. 1989. *Applied Linear Regression Models*. Homewood, IL: Irwin.
- Pan, Y. and Jackson, R.T. 2008. Ethnic difference in the relationship between acute inflammation and serum ferritin in US adult males. *Epidemiology and Infection* 136, pp. 421–431.
- Paździora, J. 1988. *Design of Underground Hard-Coal Mines*. Warszawa: PWN.
- Report 2018. *Hard coal mining in Poland*. [Online] <https://min-pan.krakow.pl/projekty/2018/07/31/raport-gornic-two-wegla-kamiennego-w-polsce-2017/> [Accessed: 2020-06-09].
- Razali, N. and Wah, Y.B. 2011. Power comparisons of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors and Anderson-Darling tests. *Journal of Statistical Modelling and Analytics* 2(1), pp. 21–33.
- Rogerson, P.A. 2001. *Statistical methods for geography*. London, UK: Sage.
- Royston, J.P. 1982. An extension of Shapiro-Wilk's W test for non-normality to large samples. *Applied Statistics* 31, pp. 115–124.
- Saługa, P. 2009. Economic evaluation and risk analysis of mineral projects (*Ocena ekonomiczna projektów i analiza ryzyka w górnictwie*). *Studia, Rozprawy, Monografie* 152, Kraków: MEERI PAS (in Polish).
- Shapiro, S.S. and Francia, R.S. 1972. An approximate analysis of variance test for normality. *Journal of the American Statistical Association* 67(337), pp. 215–216.
- Shapiro, S. and Wilk, M.B. 1965. An analysis of variance test for normality (complete samples). *Biometrika* 52(3–4), pp. 591–611.
- Sobczyk, E.J. 2008. *Analytic Hierarchy Process (AHP) and Multivariate Statistical Analysis (MSA) in Evaluating Mining Difficulties in Coal Mines*. 21st World Mining Congress – New Challenges and Visions for Mining. Kraków: Taylor&Francis Group, A Balkema Book, London.
- Sobczyk et al. 2020 – Sobczyk, E.J., Kaczmarek, J., Fijorek, K. and Kopacz, M. 2020. Efficiency and financial standing of coal mining enterprises in Poland in terms of restructuring course and effects. *Gospodarka Surowcami Mineralnymi – Mineral Resources Management* 36(2), pp. 5–30.
- Sobczyk, E.J. and Kopacz, M. 2018. Assessing geological and mining condition nuisance and its impact on the cost of exploitation in hard coal mines with the use of a multi-criterion method. *Archives of Mining Sciences* 63(3), pp. 665–686.
- Szłzak, J. 2004. Restructuring of hard coal mining in Poland in the years 1990–2002. Analysis of the effectiveness of implemented programs (*Restrukturyzacja górnictwa węgla kamiennego w Polsce w latach 1990–2002. Analiza skuteczności realizowanych programów*). Kraków: MEERI PAS (in Polish).
- U.S. Energy Information Administration (EIA) 2020. Short-Term Energy Outlook. [Online] [https://www.eia.gov/outlooks/steo/pdf/steo\\_full.pdf](https://www.eia.gov/outlooks/steo/pdf/steo_full.pdf) [Accessed: 2020-06-09].
- Weisberg, S. and Bingham, C. 1975. An approximate analysis of variance test for non-normality suitable for machine calculation. *Technometrics* 17(1), pp. 133–134.
- Wooldridge, J.M. 2013. *Introductory Econometrics: A Modern Approach* (Fifth ed.), South-Western: Cengage US Learning.

**THE ANALYSIS OF DEPENDENCE OF THE LEVEL OF OPERATIONAL COSTS AND PRODUCTION OUTPUTS UPON GEOLOGICAL AND MINING CONDITIONS IN SELECTED HARD COAL MINES IN POLAND**

**Key words**

Polish mining sector, longwall production and costs, geological and mining conditions, nuisance, AHP and regression analysis

**Abstract**

This publication presents the research aimed at developing statistical models, on the basis of which it was possible to prepare credible forecasts of unit cost and coal net output for longwalls in 5 hard coal mines in Poland. The argument has been verified that there is a dependence between the level of nuisance and the level of costs, as well as longwall production results.

A research procedure has been developed for that purpose, which aimed at developing two statistical models connecting the nuisance due to geological and mining conditions with costs and longwall production results. The multiple linear regression technique has been used to develop statistical models. The set of data taken into account in the analyses comprised 120 longwalls mined in the years 2010–2019. Two models have been developed – one for forecasting unit costs, the other for forecasting coal net output. Subsequently, the models' forecasting ability has been verified on a sample of historical data. A relative forecast error for 75% of observations has been in the range of (–25%; +37%). That result has been considered satisfactory. Subsequently, using those models, forecasts of unit costs and coal net output have been prepared for 220 longwalls planned for mining in the years 2020–2030. Those forecasts have been prepared in the stipulated ranges of geological and mining nuisance influencing mining process, by means of dedicated WUe and WUt factors. The nuisance models for forecasting purposes have been developed using the AHP (Analytic Hierarchy Process) method. The research hypothesis has been confirmed on the basis of the obtained results. An increase in the level of nuisance leads to an increase in the unit costs for longwalls and the deterioration of production results. Unit operating costs for longwalls in specific ranges of nuisance may differ by up to 30%, being in the range of 52.0–120.3 zł/Mg. Likewise, the coal daily output of longwalls may be even 22% lower, having the average level in the range of 1.89–3.61 thousand Mg/d.

ANALIZA ZALEŻNOŚCI KOSZTÓW OPERACYJNYCH  
I WYNIKÓW PRODUKCYJNYCH ŚCIAN OD WARUNKÓW GEOLOGICZNYCH I GÓRNICZYCH  
W WYBRANYCH KOPALNIACH WĘGLA KAMIENNEGO W POLSCE

Słowa kluczowe

polski sektor wydobywczy, produkcja i koszty ścian,  
warunki geologiczne i górnicze, uciążliwość, AHP i analiza regresji

Streszczenie

Publikacja prezentuje badania zmierzające do opracowania modeli statystycznych, na podstawie których możliwe było wykonanie wiarygodnych prognoz kosztu jednostkowego i wydobycia netto ścian w 5 kopalniach węgla kamiennego w Polsce. Weryfikowano tezę, że istnieje zależność pomiędzy poziomem uciążliwości a wielkością kosztów i wynikami produkcyjnymi ścian.

W tym celu opracowano procedurę badawczą prowadzącą do skonstruowania dwóch modeli statystycznych wiążących uciążliwość warunków geologicznych i górniczych z kosztami i wynikami produkcyjnymi ścian. Do skonstruowania modeli statystycznych posłużono się techniką regresji wielorakiej. Zbiór danych, które uwzględniono w analizach, obejmował 120 ścian eksploatowanych w latach 2010–2019. Powstały dwa modele – jeden dla celów prognozowania kosztów jednostkowych, drugi – produkcji węgla netto. Następnie wykonano weryfikację zdolności prognostycznej tych modeli w próbie danych historycznych. Względny błąd prognozy dla 75% obserwacji wahał się w przedziale (–25%; +37%), a jego średnia wartość dla wszystkich obserwacji nie przekraczała 5% dla obu tych modeli. Wynik ten, mimo defektów modelowania liniowego, uznano za satysfakcjonujący. Następnie przy użyciu tych modeli wykonano prognozy kosztów jednostkowych i *coal net output* dla 220 ścian planowanych do wydobycia w latach 2020–2030. Prognozy te wykonano w umownych przedziałach uciążliwości geologicznych i górniczych warunków procesu eksploatacji za pomocą wskaźników WUe i WUt. Modele uciążliwości dla celów prognostycznych skonstruowano z wykorzystaniem metody AHP (Analytic Hierarchy Process). Na bazie otrzymanych wyników teza badawcza została potwierdzona. Wzrost uciążliwości prowadzi do wzrostu kosztu jednostkowego ścian i pogorszenia wyników produkcyjnych. Zależność ta nie jest liniowa. Koszty jednostkowe ścian w poszczególnych przedziałach uciążliwości mogą się wahać nawet do 30%, mieszcząc się w przedziale 52,0–120,3 zł/Mg. Podobnie również wydobycie dobowe ze ścian może być niższe nawet o 22%, i kształtować na poziomie średnim w przedziale 1,89–3,61 tys. Mg/d.

