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The optimization of cut-off grades by means of memetic algorithms

Introduction

The determination of optimum cut-off grades is a fundamental issue in mineral extraction as it assigns the boundaries between ore and waste over time.

The traditional approach to cut-off grades is to use the break-even grade, at which revenue equals cost. This approach completely ignores the time value of money and usually leads to a sub-optimal valuation of the mineral resource.

In order to obtain maximum profit from a mineral deposit, optimum cut-off grades must be applied. Optimum cut-off grades can be achieved by a decreasing (or non-rising) order of cut-off grade policy (Lane 1964; Dowd 1976). There is an advantageous trade-off between mining at higher cut-off grades in the early years and lower cut-off grades in the later years of the life of a mine. The net cash flow (NCF) from a mining operation is a direct function

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of the sequences of cut-off grades and associated ore tonnages that define the life-of-mine production schedule. As NCF varies with these sequences, there will be a sequence, or sequences for which the optimum NCF is found. The process generally necessitates a high cut-off grade in the early years of an operation that can provide sufficient ore to satisfy the requirements of the processing plant. As time passes, the cut-off grade is lowered. Thus, the highest total discounted cash flow (DCF) is achieved.

The method used here is different to the standard cut-off process of Lane (Lane 1964) in that there is an estimate made of the amount of material below the cut-off grades used in each period that would not be mined (instead being left in-situ) in practice.

The objective of this paper is to apply memetic algorithms for determining optimal sequences of cut-off grades. A computer program was developed to this end and a case study is included to illustrate the application of the program. A sensitivity analysis was carried out for following the changes of cut-off grades and the degree of utilization of the deposit after volatile selling prices.

1. Optimization of cut-off grades

Cut-off grades are used in the mineral industry as reference grades that separate two courses of action. A major application of these grades is the classification of ore and waste. If the percentage of the useful component of the material is above the assigned cut-off grade, the material is classified as ore and could be mined at a profit. If not, the material is classified as waste and, depending on the mining method used, would be sent to waste dumps or left in-situ.

Lane (Lane 1964, 1988) must be regarded as the pioneer of formal cut-off grade theory. He provided a comprehensive economic theory and an algorithm for determining optimum cut-off grades. The work is based on subdividing the types of materials in a mining operation that are being dealt with as mineralized material, ore and mineral. The corresponding stages that the material types used are mining, mineral processing, and refining and/or marketing. However, the concepts of the mining, mineral processing and refining and/or marketing stages are different in cut-off grade terminology than in their common usage. Mineral processing and the treatment stage comprise all the activities dealing with ore, not just the metallurgical processing of ore in a mineral processing plant. The base case is as if all the mineralized material is mined as waste, and there is then the need to determine which material is ore, contributing more by being mined and processed as ore than by being mined and dumped as waste. If there is any difference in costs in the mining operations between ore and waste, this difference is related to ore, and therefore comprises part of the "ore treatment" costs.

The calculation of a single optimum cut-off grade for one particular set of conditions is trivial and the arithmetic is well within the capability of a hand calculator. The calculation of a complete cut-off grade schedule is, however, a different matter. A sequence of cut-off grades for the life of a mine has to be calculated so that the total DCF is maximum and the sequence has to be consistent in that the corresponding annual net cash flows (NCF) and present values must relate to each cut-off grade in the sequence.

Complete cut-off policies are not a normal requirement for day-to-day mining operations, but they are important for longer range mine planning.

Lane's basic expression for the DCF per unit of resource (ton) is:

$$v = (p - k) \cdot x \cdot y \cdot a - x \cdot h - m - (f + F) \cdot t \qquad (1)$$

v – discounted cash flow (\$/ton),
p – price per unit of marketed product (\$/ton),
k – marketing variable cost (\$/ton),
x – ore/material ratio (%),
y – yield from treatment (recovery) (%),
a – average grade (%),
h – mineral processing variable cost (\$/ton),
m – mining variable cost (\$/ton),
f – fixed cost (\$/(ton*year),
F – opportunity cost (\$/(ton*year),
t – time per unit of resource (year).

The inclusion of the opportunity cost in the formula generally leads to a sequence of declining cut-off grades.

Cetin and Dowd (Cetin and Dowd 2002, 2016) used genetic algorithms to find optimum cut-off grades for polymetallic deposits. They showed that genetic algorithms are more robust than many other search methods. Memetic algorithms, as a step further than genetic algorithms, promise a much better optimization performance.

2. Optimization by memetic algorithms

Memetic algorithms are considered in the class of evolutionary algorithms. Evolutionary algorithms have come a long way since the 1960s. Their roots are traced back to Holland (Holland 1975). Memetic algorithms form a combinatorial search engine that combines biological and cultural evolutions. The central philosophy of memetic algorithms is based on the very notion of Richard Dawkins (Dawkins 1976). The adjective 'memetic' comes from the term 'meme', coined by Richard Dawkins.

Memetic algorithms can be viewed as a marriage between a population-based global technique and a local search made by each of the individuals (Garg 2009).

The local search used in this research is a local improvement heuristic based on the assumption that the optimum lies in between the present state and the states of the cut-off grades from neighboring years. It is used for fine-tuning (or focusing for) the best solutions produced by the genetic algorithms.

Memetic algorithms are evolutionary algorithms particularly suited to the solution of large-scale optimization problems. They belong to the class of probabilistic algorithms but are very different from random algorithms as they combine directed and stochastic searches. Another important property of evolutionary search methods is that they maintain a population of potential solutions. Memetic algorithms can also easily escape from local optima by using genetic operators.

Cetin (Cetin 2016) used memetic algorithms in the optimization of cut-off grades under uncertain market conditions. He argued that the selling price of a mineral must be stochastically added to the algorithm.

The steps of an algorithm of a computer program to be developed for memetic algorithms are as follows:

1. Encode solution space. Encoding chromosomes in order to search for solutions by using binary bits.
2. Set the number of the population (chromosomes) and size generation. Set the crossover and mutation rates.
3. Initialize population.
4. Apply genetic algorithms.
5. Apply local search.
6. Apply final local search to the fittest person (chromosome).

3. Depletion

Depletion is the amount of material depleted regardless of whether it is excavated or left in-situ. The depletion rate is the rate of a mineral deposit which is to be depleted annually. Part of the depletion is sent to the concentrator as ore, part of it is mined out and dumped as waste material and part of it is left in-situ.

Production or the production amount is the part of material that is extracted and sent to mineral processing facilities.

The difference between the depletion and the production in a mine is the material below the used cut-off. Depending on the characteristics of mineral deposits, some of the blocks that have an average grade less than the determined cut-off grade are left in-situ, some of them are excavated and dumped as waste material.

Depletion or the depletion rate does not refer to the portion or the rate of deposit to be excavated. It refers to the portion or rate of deposit that is depleted, regardless of whether it is excavated or not.

4. Mining cost

The cost of extracting that takes place in the life of a mine is considered to be a variable mining cost and must be applied to all the drilling, blasting, excavating, loading, hauling and dumping.

In Lane's theory, variable mining costs are to be applied to all excavations, regardless of whether they are sent to the concentrator or dumped. However, for some of the blocks that are to be left in-situ, variable mining costs should not be an issue.

In the determination of cut-off grades, if all parts of a mineral deposit are considered to be accessed immediately, that is, if there are no access constraints, the variable mining costs are to be applied to the production rates. Conversely, if all the blocks (including those with grades less than the cut-off grade) must be excavated because of access constraints, the variable mining costs need to be applied to the depletion rates.

It is well known from practice that some of the blocks are left in-situ. Leaving any portion of a mineral deposit that is not considered as ore after economic appraisals unexcavated is always preferable to excavating and dumping as the latter generates further rehabilitation costs. The parts of the deposit that have an average grade less than the cut-off grade determined should be left in-situ whenever possible.

In short, the problem can be written as whether the variable mining costs need to be applied to the depletion rate of the deposit (total excavation) or to the production rate of the deposit (total amount of ore sent to the concentration units).

The application of variable mining costs to the depletion rate or to the production rate is not realistic. The approach must be that the truth lies in between. The true or applicable approach is that the variable mining cost should be applied somewhat in between the depletion rate and the production rate. In order to solve this problem, the proportion of depletion that is to be left in-situ each year must be known. However, the application of variable mining costs to the depletion rate is closer to open pit mining and the application of variable mining costs to the production rate is closer to underground mining.

Kenneth Lane is considered as a pioneer of optimum cut-off grades theory. Before him, the breakeven grade had been used as the cut-off grade to divide a mineral deposit into ore and waste. However, according to the algorithm he developed (Formula 1), the variable mining cost is applied to the total volume of a designed pit, i.e. all the mineralized material. However, the amount of material that has a grade lower than the determined cut-off grade should be left in-situ for as long as it is possible; however, because of access constraints, it is not possible for most of the blocks in open pit mines.

As it is known from practice, depending on the characteristics of mineral deposits, that some of the blocks with an average grade less than the determined cut-off grade are left in-situ, and some of them are excavated and dumped as waste material. Naturally, variable mining costs should not be applied to the blocks of a mineral deposit that are left in-situ.

The main problem arises from the fact that the part of a mineral deposit that is to be left in-situ cannot be known before a complete mine scheduling takes place, and mine scheduling cannot be done before the determination of cut-off grades. They must be done simultaneously. However, it is a large scale optimization problem that could not yet be solved. Instead, a probabilistic approach can be used in this context. That is, the part of the depletion rate of a mineral deposit in a time period which is to be left in-situ can be determined probabilistically. Probabilistically determined partial depletion cost is to be added to the Lane's (Lane 1964) algorithm in this context. The DCF algorithm per unit of resource should then be rewritten as:

$$v = (p - k) \cdot x \cdot y \cdot a - x \cdot h - (x + z) \cdot m - (f + F) \cdot t \tag{2}$$

z – (material to be dumped)/material ratio.

The determination of the portion of depletion that is to be dumped after excavation can be searched for by means of statistical tools. In open pit mines, because of access constraints, in the early years of a mine life, in general, all blocks are to be extracted regardless of being ore or waste. In the later years, more blocks can be left in situ, although most of the blocks are excavated and dumped. For this reason, exponential distribution is considered as a good tool for this subject in the case of open pit mines.

The probability density function of an exponential distribution is used to find the proportion of the depletion rate over the production rate that is to be left in situ. As a result, the inverse probability density function is to be applied as the proportion of the depletion rate over the production rate that is to be excavated and dumped.

The probability density function (PDF) of an exponential distribution is given in Figure 1 and Formula 3.

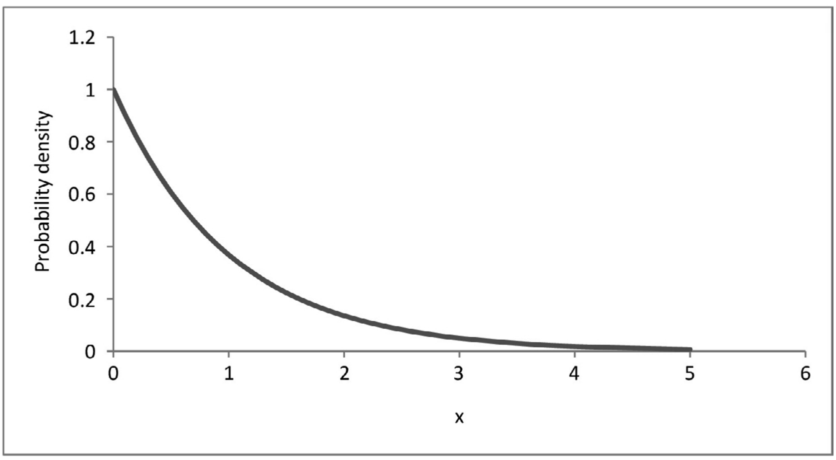

Fig. 1. Probability density function of an exponential distribution

Rys. 1. Funkcja gęstości prawdopodobieństwa rozkładu wykładniczego

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{3}$$

↳ f – the proportion of the material below the cut-off used in each period that remains unmined,
λ – rate parameter, used 1,
x – the number of periods of mine life remaining at the start of the period being evaluated.

5. Rehabilitation cost

Another cost that is to be added to the cut-off grades algorithm is reclamation or rehabilitation costs originating from the fact that the blocks that are excavated but dumped as waste material and also tailings from the treatment facility incur some additional cost of rehabilitation. Gholamnejad (Gholamnejad 2008, 2009) mentioned the rehabilitation cost and included it in Lane's algorithm. He added the rehabilitation cost into Lane's formulations of net present value determination.

Rehabilitation is the treatment of disturbed land for the purpose of establishing a stable environment. The aim of rehabilitation is to ensure that mining activities are designed to minimize or mitigate adverse environmental and social impacts and create a self-sustaining ecosystem.

6. Application of memetic algorithms to cut-off grade optimization with rehabilitation cost

Many cut-off grades optimization problems have huge numbers of local optimum values which are widely separated from the global optimum point and from each other. Stochastic search methods can easily fail to find the global optimum point for such problems. The real challenge in such problems is finding solutions close to the global optimum point for a restricted time. Memetic algorithms are more robust in this context than many other existing search methods.

The encoding and evaluation processes used in this paper for the application of memetic algorithms for cut-off grade optimization are described below:

6.1. Encoding

The encoding of an individual for the optimization of a single cut-off grade for an ore deposit is straightforward. If the binary representation is used, the length of a string will be long. A 5-bit string can represent $2^5 = 32$ cut-off grades. To derive real values from the binary code (i.e., mapping) of the string, the formula is:

$$X = X_{\min} + \frac{X_{\max} - X_{\min}}{2^L - 1} \cdot Y \tag{4}$$

- X – the mapping value,
- $X_{\min}$ – the minimum cut-off grade to be searched for,
- $X_{\max}$ – the maximum cut-off grade to be searched for,
- L – the length of the binary string,
- Y – the value of binary representation.

The value of the binary representation for a 5-bit string would be an integer between 0 for string 00000 and 31 for string 11111.

The application of memetic algorithms to the solution of the optimum cut-off grades problem requires a crucial increase in the length of the string. Because in each year of mine life there might be a different optimum cut-off grade, there should be different genes in the same string. If the mine life is 20 years, the string should be composed of 20 genes, each with a length of 5-bits, making the total length of the chromosome 100.

6.2. Evaluation

In memetic algorithms, every individual is assigned a fitness value depending on its performance. In cut-off grade optimization, the objective function is the maximum DCF. The higher the DCF, the better the individual is.

It is very clear from this work, and works done by others, that the maximum DCF is generally achieved by a declining cut-off grade policy. In other words, the mining operation starts with a relatively high cut-off grade that declines gradually over the life of the mine.

The local search used in this work is deterministic in nature – the program searches all of the possible values one by one. All the cut-off grades in the sequence of all the survived chromosomes (population) and of the best chromosome are changed explicitly.

The rehabilitation cost is added to the algorithm explicitly, i.e. the user must enter the data. For the partial depletion rates (the part of depletion applied to the variable mining cost), the probability density function is used.

After the inclusion of different depletion rates and the rehabilitation cost in the algorithm, Lane's (1964) equation for cut-off grades becomes:

$$v = (p - k) \cdot x \cdot y \cdot a - x \cdot (h) - (x + z) \cdot (m + r) - (f + F) \cdot t \tag{5}$$

- r – variable rehabilitation.

A computer program has been developed and applied to implement the developed memetic algorithms.

7. Case study

A case study is included here to illustrate the application of the program for determining optimal cut-off grades by means of memetic algorithms. The case study concerns a copper deposit. The grade-tonnage distribution and the technical and economic data for the mine are shown in Table 1 and Table 2.

Table 1. Grade-tonnage distribution for the copper deposit

Tabela 1. Rozkład granulometryczno-tonażowy złoża miedzi

Grade (%)	Tonnage (*million tons)
0.00–0.15	0.17
0.15–0.30	2.24
0.30–0.45	3.60
0.45–0.60	9.28
0.60–0.75	18.11
0.75–0.90	16.19
0.90–1.05	10.14
1.05–1.20	7.02
1.20–1.35	4.27
1.35–1.50	0.91
1.50–1.65	0.40
1.65–1.80	0.24
1.80–1.95	0.15
1.95–2.10	0.10

The computer program developed for this subject was written using C++ codes. The program terminates in 25 CPU seconds. The results are given in Table 3.

Cut-off grades and depletion rates in the above table are in decreasing (non-rising) order. Cut-off grades start with 0.57% and decrease slowly to 0.03%. The decreasing order of the cut-off grades can be traced from Figure 2. The total DCF is $1,648,350,000 and the total production is 64.94 million tons. Memetic algorithms form a very robust search engine since

Table2. Technical and economic data

Tabela 2. Dane techniczne i ekonomiczne

Description	Value
Lower limit of cut-off grades	0
Upper limit of cut-off grades	0.93
Mining capacity (tons per day)	5,000,000
Mineral processing capacity (tons per year)	4,000,000
Marketing and/or refining capacity (tons per year)	40,000
Selling price (dollars per ton)	9,730
Marketing and/or refining cost (dollars per ton)	1,500
Recovery rate (%)	92
Variable mining cost of mined material (dollars per ton)	2.4
Variable rehabilitation cost of mined material (dollars per ton)	0.8
Variable concentration cost of processed material (dollars per ton)	9.6
Fixed costs (dollars per year)	2,500,000
Discount rate (%)	10
Population size (number of individuals in the population)	1,000
Number of generations	1,000
Crossover rate (%)	50
Mutation rate (%)	60

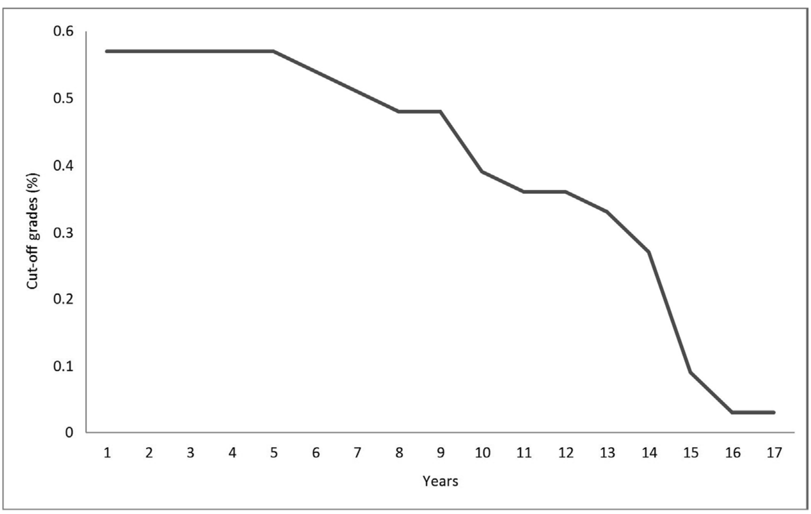

Fig. 2. Cut-off grades for the copper deposit

Rys. 2. Klasy odcięcia dla złoża miedzi

Table 3. Output file for the memetic algorithms. The total Discounted Cash Flow is $839.85 Million

Tabela 3. Plik wyjściowy algorytmów memetycznych; całkowity zdyskontowany przepływ pieniężny wynosi 839,85 mln USD

Year	Cut-off grades (%)	Discounted cash flow ($m)	Net cash flow ($m)	Depletion rate (Mt)	Production rate (Mt)	In-situ rate (kt)	Marketing rate (kt)
1	0.57	195.88	215.47	4.90	4.00	0	33
2	0.57	178.07	215.47	4.90	4.00	0	33
3	0.57	161.89	215.47	4.90	4.00	0	33
4	0.57	147.17	215.47	4.90	4.00	0	33
5	0.57	133.79	215.47	4.90	4.00	0	33
6	0.54	120.17	212.89	4.76	4.00	0	32
7	0.51	107.86	210.20	4.62	4.00	0	32
8	0.48	96.75	207.40	4.48	4.00	0	32
9	0.48	87.96	207.40	4.48	4.00	0	32
10	0.39	77.91	202.07	4.27	4.00	0	31
11	0.36	70.36	200.76	4.22	4.00	1	31
12	0.36	63.97	200.76	4.22	4.00	2	31
13	0.33	57.75	199.38	4.18	4.00	3	30
14	0.27	51.88	197.00	4.11	4.00	6	30
15	0.09	46.10	192.58	4.01	4.00	1	30
16	0.03	41.87	192.38	4.00	4.00	1	30
17	0.03	8.96	45.29	0.94	0.94	0	7

they can easily escape from a local optimum point by means of crossover and mutation tools, and its natural selection environment and the optima from genetic algorithms program are re-evaluated by means of a local search.

To illustrate the sensitivity of the cut-off grades, the total DCF and the total production to the fluctuations in the copper price, a sensitivity analysis was carried out. Table 4 shows the changes in the cut-off grades when the copper price deviates by ±10% and ±20% from the value of $9,730 per ton.

It can be seen from Table 4 and Figure 3 that the cut-off grades are not very sensitive to fluctuations in the copper price. For all the prices, the cut-off grades start with 0.57% and end up with 0.03%. Naturally, there are some differences for the years in between as otherwise they would be identical, which is not expected.

Although it is not easy to trace from Table 4 and Figure 3, the cut-off grades decrease slightly with increasing selling prices, which is to be expected. It is a rule of thumb for

Table 4. Cut-off grades as a function of different copper prices

Tabela 4. Klasy odcięcia w funkcji różnych cen miedzi

Cut-off grades (%)					
Year	Selling price (–20%) ($7,784/ton)	Selling price (–10%) ($8,757/ton)	Selling price ($9,730/ton)	Selling price (10%) ($10,703/ton)	Selling price (20%) ($11,676/ton)
1	0.57	0.57	0.57	0.57	0.57
2	0.57	0.57	0.57	0.57	0.57
3	0.57	0.57	0.57	0.57	0.57
4	0.57	0.57	0.57	0.57	0.57
5	0.57	0.57	0.57	0.57	0.57
6	0.57	0.57	0.54	0.57	0.54
7	0.54	0.54	0.51	0.51	0.51
8	0.51	0.51	0.48	0.51	0.48
9	0.48	0.48	0.48	0.48	0.45
10	0.45	0.42	0.39	0.33	0.42
11	0.42	0.39	0.36	0.30	0.39
12	0.36	0.39	0.36	0.24	0.36
13	0.30	0.39	0.33	0.18	0.21
14	0.27	0.33	0.27	0.18	0.12
15	0.21	0.30	0.09	0.15	0.12
16	0.03	0.06	0.03	0.09	0.09
17	0.03	0.03	0.03	0.03	0.03

Table 5. Total DCF and total production as a function of different copper prices

Tabela 5. Całkowity DCF i całkowita produkcja jako funkcja różnych cen miedzi

	Total DCF	Total production (tons)
Selling price (–20%) ($7,784/ton)	$1,161,257,008	64,328,340
Selling price (–10%) ($8,757/ton)	$1,404,919,351	64,351,705
Selling price ($9,730/ton)	$1,648,350,000	64,941,630
Selling price (10%) ($10,703/ton)	$1,891,287,367	65,188,320
Selling price (20%) ($11,676/ton)	$2,135,613,839	65,193,917

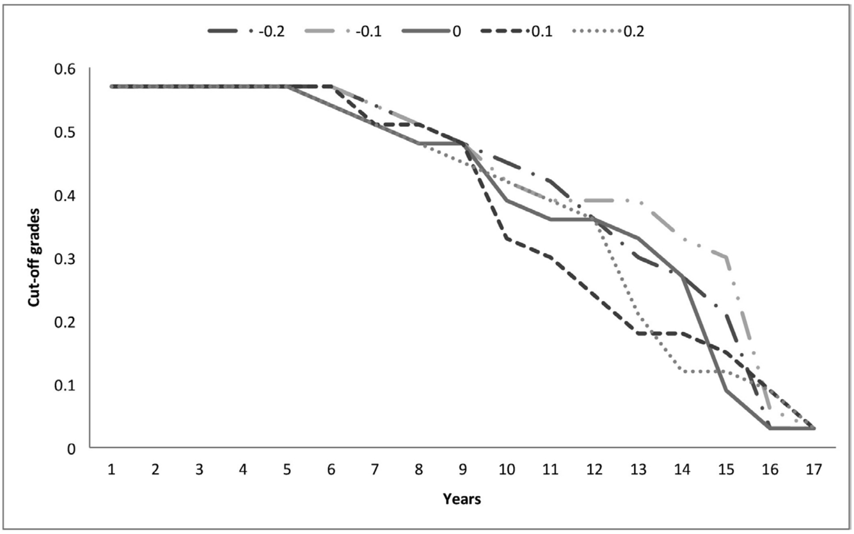

Fig. 3. Cut-off grades for different selling prices for copper

Rys. 3. Klasy odcięcia dla różnych cen sprzedaży miedzi

optimum cut-off grades that in order to trace a decrease or increase in cut-off grades, one should check the production rates. Because an increase in production rates is only possible with a decrease in cut-off grades, an increase in the total production suggests a decrease in the cut-off grades and vice versa. It can be traced from Table 5 and Figure 4 that since the total production increases slightly with increasing selling prices, the optimum cut-off grades decrease slightly.

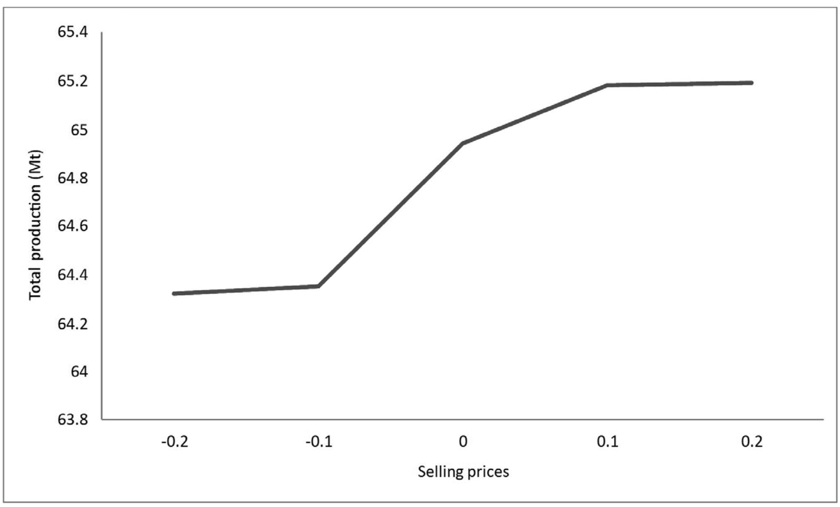

Fig. 4. Total production for different copper prices

Rys. 4. Całkowita produkcja dla różnych cen miedzi

Table 5 also shows that the total DCFs increase with an increase in copper prices and decrease with a decrease in those prices.

Conclusions

It is very clear from this research, and work done by others, that maximum discounted cash flow can generally be achieved by a declining cut-off grades policy. In other words, a mining operation should start with a relatively high cut-off grade that declines gradually over the life of the mine.

The use of evolutionary algorithms in cut-off grade optimization has been gaining popularity in recent years. The memetic algorithms method is a very robust search engine. It is as a step further on from genetic algorithms and is a promising optimization tool. The crossover, mutation and natural selection behavior of the method ensures it escapes from a local optimum point and a further local search improves the optimum further.

The results from the case study show that evolutionary algorithms are very useful in searching for optimum cut-off grades.

The main contribution of this work to cut-off grade optimization is extending Lane's cut-off grades theory into use of a more realistic depletion rate applicable to variable mining costs and the addition of rehabilitation costs. The work described in this paper is the first application of memetic algorithms for cut-off grade optimization in this context.

The parts of a mineral deposit that are not profitable should not be mined. However, because of access constraints, they have to be excavated and dumped. If all the blocks of a mineral deposit are to be excavated for that reason, then the variable mining cost (the cost of excavation, haulage and dumping) is applied to the depletion rates. However, if some blocks can be left in-situ (which is preferable), then the variable mining cost is to be applied to the partial depletion rates. Although this is always the case in practice, it is constantly overlooked. Mining cost must not be the subject for the portion of the depletion that is left in-situ. The partial depletion rates can be searched for by means of statistical tools. The inverse probability density function is used as the portion of the depletion that is to be excavated and dumped.

The rehabilitation cost is to be applied to the portion of the depletion that is to be extracted and dumped. The unprofitable blocks that are excavated and dumped incur some rehabilitation costs. The related rehabilitation costs are included in the algorithms. The overall rehabilitation cost that will mainly be carried out following the closure of the mine is beyond the scope of this work.

The fact that the sensitivity analysis for fluctuating copper prices did not significantly change the optimum cut-off degrees policy demonstrates the robustness of the work done in this study.

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THE OPTIMIZATION OF CUT-OFF GRADES BY MEANS OF MEMETIC ALGORITHMS

Keywords

memetic algorithms, optimization, cut-off grades, depletion rate, rehabilitation cost

Abstract

Cut-off grades optimization is a fundamental issue for mineral deposits. A cut-off grade is any grade that is used to separate two courses of action; to mine or not to mine, to process or to dump. In order to achieve the maximum discounted cash flow, generally a decreasing order of cut-off grades schedule takes place. Variable mining costs are applied to the extracted material, not to all of the depletion rate as some of the depletion can be left in-situ. Because of access constraints, some of the blocks that have an average grade less than the determined cut-off grade are left in-situ, some of them are excavated and dumped as waste material. The probability density function of an exponential distribution is used to find the portion of the material below the cut-off used that is left in situ. The parts of a mineral deposit that are excavated but will be dumped as waste material and tailings of ore incur some additional cost of rehabilitation. The method of memetic algorithms is a very robust optimization tool. It is a step further from the genetic algorithms. The crossover, mutation and natural selection behavior of the method ensures it escape from a local optimum point, and a further local search improves the optimum further. This paper describes the general problem of cut-off grades optimization, outlines the use of memetic algorithms in cut-off grades optimization and further extension of the method including partial depletion rates and variable rehabilitation cost. This paper is the first application of memetic algorithms to cut-off grades optimization in this context.

OPTYMALIZACJA WARTOŚCI GRANICZNEJ ZA POMOCĄ ALGORYTMÓW MEMETYCZNYCH

Słowa kluczowe

algorytmy memetyczne, optymalizacja, wartości graniczne,
wskaźnik sczerpania, koszty rekultywacji

Streszczenie

Optymalizacja wartości granicznej (brzeżnej) jest fundamentalną kwestią dla złóż kopalin. Wartość graniczna jest to ocena, która służy do oddzielenia dwóch kierunków działania; wydobywać lub nie wydobywać, przetwarzać lub składować. Aby osiągnąć maksymalny zdyskontowany przepływ pieniężny, na ogół ma miejsce malejąca kolejność harmonogramu wartości granicznej. Do wydobytego surowca stosuje się zmienne koszty, a nie do całego wskaźnika zubożenia (sczerpania), ponieważ część surowca można pozostawić in situ. Ze względu na ograniczenia dostępu do surowca mineralnego, niektóre bloki, które mają średnią wartość mniejszą niż określona wartość graniczna, są pozostawiane na miejscu (*in situ*), niektóre z nich są wydobywane i składowane jako materiał odpadowy. Funkcja gęstości prawdopodobieństwa rozkładu wykładniczego jest stosowana do znalezienia części materiału poniżej wartości granicznej, która pozostaje na miejscu. Części złoża mineralnego, które są wydobywane, ale będą składowane jako surowiec odpadowy jak również odpady rudy, wiążą się z dodatkowymi kosztami rekultywacji. Metoda algorytmów memetycznych jest bardzo solidnym narzędziem optymalizacyjnym. Jest o krok dalej od algorytmów genetycznych. Zachowanie krzyżowania, mutacji i selekcji naturalnej tej metody, zapewnia jej odejście z lokalnego punktu optymalnego, a dalsze lokalne poszukiwania jeszcze bardziej poprawiają optimum. W artykule opisano ogólny problem optymalizacji wartości granicznej, przedstawiono zastosowanie algorytmów memetycznych w optymalizacji wartości granicznej oraz dalsze rozszerzenie metody o współczynniki częściowego ubytku i zmienne koszty rekultywacji. Niniejszy artykuł jest pierwszym zastosowaniem algorytmów memetycznych do optymalizacji wartości granicznej dla złóż kopalin.