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The Aumann-Maschler bargaining set as a criterion for the division of mining royalty

Introduction

A taxation regime (fiscal system) is generally a set of tax regulations within a given country forming a coherent whole, both in the legal and the economic sense. A properly constructed taxation regime should be efficient, flexible, stable, fair, neutral, convenient, effective, transparent and certain. With regard to mining and, more broadly, raw material activities, the taxation regime is an individual solution set at the national level. It is usually based on the country's mining traditions, the volume and number of deposits, and the size and type of mineral resources; it also takes into account the ownership rights of mineral deposits and their global importance. The taxation regime, in addition to the political and economic factors present in a given country, can be an important decision-making factor for investors who want to undertake and implement raw material projects (Yunis and Ali-akbari 2022). The mining sector, which bases its activity on mostly non-renewable deposits

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of minerals, requires a separate and specific fiscal approach other than the approaches normally adopted for other sectors of the economy. The system of tax burdens (depreciation, costs of exploration and mining licenses, mining lease (in Poland, so-called mining usufruct), mining levies, income tax and local taxes etc.) should be determined and defined in such a way as to take into account the complex status of the raw material projects. Mineral deposits, defined as the basic assets of mineral raw material projects, are specific natural objects, and their characteristics include: high investment risk, the universal non-renewability, the uneven distribution of deposits in geographical terms, and in most cases, limited reserves.

Mining, by generating surplus value, creates wealth. This potential is a strong incentive for private companies to explore, develop and then exploit deposits. The driving force behind the operation of mining companies is sometimes the pursuit of profit, although it is often at odds with the goals and objectives of the national government. The conditions under which private entities have access to state owned deposits are regulated by internal decisions, and the implemented policies of many states concerning raw materials e.g. through taxes imposed on the mineral processing sector aimed at promoting various social, economic, development or environment-related goals and is often imposed in accordance with the dominant processes and political views of the parties currently in power (Otto et al. 2006; Hein and Cecot 2017).

Taxation of mining activities is an approach that has been practiced since ancient times. Taxes and other levies for exploration, concessions, lease, production and other activities provide central and local governments with revenues (own revenues) that can later be used in a variety of ways, including those transferred to the society in the form of subsidies for education or health care etc. It seems reasonable that at least part of the funds obtained in this way should be spent on the management of natural resources and environmental protection, as well as research and development promoting innovation and reduced resource consumption (OECD 2011). The fiscal categories of burdens imposed on mining entrepreneurs commonly used in the world are very extensive. Some of the liabilities are in the form of income levies of the *Corporate Income Tax* (CIT) category, some are of a customs nature, protecting the internal market and/or increasing state budget revenues, others result from real estate taxation, wages, VAT rates, lease, land use change and other factors. Most important in the category of deposits and their resources management is mining royalty and this type of duty will be the subject of further analysis in this paper.

The word *royalty* has its roots in English common law and refers to the institution of the British Crown, where "rights or privileges" belonging to the Crown were defined at the beginning of the fourteenth century (Hein and Cecot 2017). Today, the meaning of *royalty* is similar to the category of rent; in the past, it was generally part of the output given to the crown in exchange for the privilege of mining. The term *royalty*, as well as its legitimacy and functionality, has remained the subject of scientific debate from the very beginning (Hotelling 1931); however, independently of this dispute, *royalty* is considered one of the basic categories of levies paid by mining entities, and it is difficult for the legislator to deter-

mine its appropriate and justified level. The specificity of mineral resources, because they are usually depleted irretrievably, is different from other natural resources (Dasgupta and Heal 1974; Dasgupta and Stiglitz 1981; Adelman 1990; Steen 2006; Aguilera et al. 2009; Kharitonova et al. 2013; Scholz and Wellmer 2013; Northey et al. 2018; Tilton et al. 2018; Castillo and Eggert 2020), and the renewal of the resource base requires the exploration and documentation of deposits, often requiring new environmentally and socially sustainable approaches (Vincent et al. 1997; Prior et al. 2012; Dubiński 2013; Höök and Tang 2013; Henckens et al. 2016a, b; Calvo et al. 2017; Schmidt 2019; Seifi 2021). Therefore, establishing a dependency between resource depletion (capital loss) and the level of *royalty* seems to be a justified decision (Anderson 1997a, b). Currently, apart from various doubts and ambiguities on the level of *royalty*, it is considered a levy commonly used in the mineral resources management system (Boadway and Flatters 1993; Tilton 2004; Baccareza and Humanez 2014; Hein and Cecot 2016), and the very specificity of *royalty* has been the subject of research in numerous publications (Heaps and Helliwell 1985; Neyra 2011; Paredes and Riviera 2017; Otto 2018), and in a comprehensive approach, this has been compiled in a study by Otto et al. (2006).

The feature differentiating *royalty* in individual countries is the assessment base. The three most common systems in use are based on:

1. Tonnage or unit based on extracted mineral or tonnage/volume of produced raw material.
2. The value-based of production sold (*ad valorem*).
3. Profit or income-based system (sales value of production less costs).

Combinations of these are also possible and as such are used, for example, in India and Australia. Apart from the assessment base criterion of *royalty*, this levy should undoubtedly be transparent, fair and objective (Chadha and Kapoor 2022). The level of taxes and levies in the mining industry has been widely discussed (Baunsgaard 2001; Tilton 2004), but the responsibility of the entity imposing and controlling the *royalty* (government) is to adjust the level of levies so that, on the one hand, they are a source of compensation for the local, regional or national communities for irretrievably lost resources and environmental damage, and on the other hand, these imposed levies are of a level acceptable to investors so as to not discourage business activity (Sorley 1889; Gajigo et al. 2012). It happens that the level of *royalty* is politically motivated, modelled by crises (Corrick et al. 2020; Darvas and Martins 2022). Regardless of the circumstances, high-level *royalty* may be the simplest and most highly effective form of protection of particularly valuable mineral resources.

1. Mining royalty in Poland

In the past, a so-called *regale* or *królewszczyzna* was imposed on miners in Poland, and the king had the exclusive right to utilize the minerals deposited in royal, church and

secular lands. When an area used for mining was leased, the king collected the tithe in kind (a part of the raw material amounting to about 10% of the output) or a levy called *olbora* (lease rent) (Szamałek 2001). During the period of the Partition of Poland, depending on the partitioning subordination of territories, fiscal regulations in Poland were determined by the legislation of the partitioning powers. After Poland regained independence, the reform of 1930 introduced mining law that divided mineral deposits into four categories, including those reserved for the state. After the Second World War, mining royalties did not exist in Polish legislation and the resources of the deposits remained the property of the state without sovereignty rights. Currently, in Poland, a unitary state, all categories of levies are set at the central level and the mining royalty as a regulatory levy was reintroduced in the amendment to the Mining and Geological Law in 1991. One of the most important reasons for resuming the collection of the mining royalties was the need to protect the deposit, and mining royalties, classified as costs, were to stimulate the rational management of the deposit (the rarity of mineral results in a higher unit amount of the royalty for its extraction). In addition, the mining royalty is a form of internalization of the benefits derived by the user as a result of the extraction of natural mineral resources and their proper allocation by the local or central government. These and other multiple functions of the mining royalty in Poland can be aggregated into several categories (Galos and Smakowski 1993; Szamałek 2001, 2005): restoration, compensatory, stimulating, valorization, protective, fiscal, preventive and repressive.

The standard of the mining royalty in Polish legislation is the fee calculated in relation to the tonnage or volume of the extracted mineral. The user of the deposit pays the mining royalty to the bank accounts of the royalty creditors, i.e. National Fund for Environmental Protection and Water Management (40%) and the local governments (communities) (60%). Pursuant to the regulations, if the mining activity is performed in more than one district, one county or one voivodeship, the royalties constitute the income of the specified units, proportional to the amount of extracted mineral or the surface area covered by the mining activity. To determine the service levies, the rates contained in the announcement of the Minister of the Environment on the rates of royalties for a given year shall apply. Total revenues from mining royalties in Poland amounted to PLN 370–380 million annually in the period 2017–2021.

2. Aumann-Maschler bargaining sets – a theoretical outline

The proposal for the division of the revenues from the mining royalties, presented in the further part of this paper, was calculated based on Aumann-Maschler bargaining sets (Aumann and Maschler 1961). This approximation is based on the criteria for solving multiplayer games referring to a bargain situation in which it is necessary to divide a specific resource among many consumers. It has to be assumed that the possible outcomes of the division are

more favourable than no agreement. Two basic approaches to the bargain problem are postulated in the literature on the subject:

1. Axiomatic model (normative, static).
2. Strategic model (dynamic).

The axiomatic model, which is used in this paper in further considerations, does not describe the bargaining procedure, i.e. the rules and course of negotiations, but only analyses the possible outcomes. Taking into account the strategic actions of the players and their preferences, and based on the established axioms, it gives a clearly defined solution. The problem of fair division is the subject of many years of research, but due to the degree of complexity and the subjectivism of participants towards parceling, it remains equivocally defined and unresolved. A fair division from the perspective of one or a group of participants does not mean that the other participants in the game will consider it fair. The question of fair distribution in the axiomatic approach was raised by the game theory pioneer Nash (1950), the aforementioned Aumann and Maschler (1961), Rapoport (1970a), Rubinstein (1982), Anderson (1994), Thomson (1994), and Brams and Taylor (1996).

The idea of the Aumann-Maschler axiomatic solution comes down to a specific division of some resource between the bargain participants. In the formal language of mathematics, it is an n-person game and for its correct description it is convenient to use the so-called "characteristic function". A game in the form of a characteristic function is described by a set of $n = \{1, 2, \ldots, n)$ players and function v, which assigns a number $v(S)$ to every subset S (coalition) $S \subseteq N$. Other players, not joining the coalition are considered so-called "anti-coalition" $(N - S)$. Size $v(S)$ is the win value (expected value, price or value of the game) that the players belonging to coalition S will achieve together if they enter into it, regardless of the moves of the other players in the game (Rapoport 1970b; Luce and Raiffa 1989; Thomas 2003; Tchantho et al. 2012). The calculated value of the function for each possible coalition, i.e. each possible subset of players, is the said characteristic function of the game. The value of the coalition joined by no-one (empty) is 0, while an alliance of two or three members of a division always provides a payoff at least equal to the sum of individual payoffs, although coordination of moves generally leads to higher payoffs. According to the concept of Aumann-Maschler (1961), one should not expect a grand coalition (all participants of the bargain) composed of all interested parties but arrangements with smaller numbers are possible; however, from the point of view of group rationality, only a grand coalition can lead to sharing the entirety of the available resource and ensure the highest total payoffs. Entities participating in the bargain can communicate with each other (form coalitions), and the result of negotiations, own preferences as well as individual bargaining power is a stable payoff vector $x = (x_1, x_2, ..., x_n)$. The stability of the solution in accordance with Aumann-Maschler's idea results from the fact that each objection can be presented with an effective counter objection by any member of the coalition, and thus the solution is the so-called "bargaining set". Formally, Aumann-Maschler's concept of bargaining sets requires the payoff vector $x = (x_1, x_2, ..., x_n)$ in a given structure to be rational, i.e. it must meet the criteria of individual $\forall_{i \in N} x_i \geq v(\{i\})$ and collective $\Sigma_{i \in N} x_i \geq v(N)$ rationality. The stability of the payoff vector

requires (reasoning for two players is presented) the following conditions: if two players K, $L \in S_j$ then K formulates an objection against L such that: if there is such a coalition, $S \subseteq N$ and the payoff vector is $y = (y_1, y_2, ..., y_n)$ in which:

◆ objection 1 – K belongs to S but L does not belong to S;
◆ objection 2 – $\forall_{k \in S} y_k > x_k$;
◆ objection 3 – $\Sigma_{k \in S} y_k = v(S)$;

then L has an effective counter-objection against K if there is another coalition of M and the payoff vector is $z = (z_1, z_2, ..., z_n)$ such that:

◆ counter-objection 1 – L belongs to M but does not belong to M;
◆ counter-objection 2 – $\Sigma_{k \in M} z_k = v(M)$;
◆ counter-objection 3 – $\forall_{k \in M} z_k \geq x_k$;
◆ counter-objection 4 – $\forall_{k \in M \cap S} z_k \geq y_k$;

Here, the payoff vector $x = (x_1, x_2, ..., x_n)$ is stable in the coalition structure S_n if each objection can be opposed with an effective counter objection. The set of all stable payoff vectors in any coalition structure is then the Aumann-Maschler bargaining set of this structure (Aumann and Maschler 1961).

3. The application of the Aumann-Maschler bargaining set for the mining royalty division – case study

Aumann-Maschler's stable bargaining sets methodology was used for the case study of Racibórz II-Reservoir 5 sand and gravel deposit. This deposit is located in the Śląskie Voivodeship, the Silesian Upland on the western part of Rybnik Plateau. It is actively mined and covers an area of 35.78 ha located within three communes. The basic surface and resource indicators as well as the production volume in 2021 are presented in Table 1.

Racibórz II-Reservoir 5 deposit has been exploited since 2012, and in the years 2018––2020, the exploitation was conducted in the three districts and reached the extraction level in the range of 290,000–303,000 tons of gravel. In 2021, mining in the Kornowac district

Table 1.   Reserves and exploitation characteristics of Racibórz II-Reservoir 5 deposit in 2021
             (rounded to the nearest thousand, PGI-NRI 2022)

Tabela 1.   Zasoby i charakterystyka eksploatacyjna złoża Racibórz II-Zbiornik 5 w roku 2021

District	Area (ha)	Share of area (%)	Reserves (tons)	Mining output (tons)
Racibórz	5.70	15.9%	293,500	20,000
Kornowac	8.28	23.1%	426,500	0
Lubomia	21.80	61.0%	1,126,000	155,000
Total	35.78	100.0%	1,846,000	175,000

was discontinued and extraction in the other districts was reduced, which resulted in a decrease in production to approximately 175,000 tons. Mining based on an exploitation concession in each district requires agreement and the positive opinion of the district authority, although decisions of authorities in this matter do not have to be unanimous. It is possible that one or more district authorities do not consent to mining. The decision-making problem of the district authorities then boils down to defining their position and deciding on another form of land use. On the one hand, it is consented to conduct mining activities in its area; on the other hand, it can be any other form (e.g. agricultural, recreational, construction, etc.) of land use. Mining is generally not an activity preferred by local communities and sometimes receives strong opposition (Prno and Slocombe 2012; Moffat and Zhang 2014; Radwanek-Bąk 2018; Komnitsas 2020; Cruz et al. 2021; Heffron et al. 2021; Tost et al. 2021); however, viewed from another angle, the exploitation of minerals provides access to mineral resources necessary for the production of many basic everyday goods, enables civic development, increases the level of prosperity and also improves the condition of infrastructure by supplying the necessary road and construction materials.

In the analyzed case, the essence of the proposal was that mining is possible in any alliance of the districts (mining sanctioned by the district authorities) and will be performed regardless of their number. It is thus possible to start exploitation in the portions of the deposit belonging to, for example, only one of the districts; however, only a comprehensive deposit development in all districts will allow for the full use of resources and will meet the requirement of the rational use of natural resources. The analysis was conducted for hypothetical production levels, depending on the coalition. It was assumed, regardless of the size and composition of the coalition, that two raw materials would be mined, namely 0–2 mm washed sand and 2–16 mm gravel. The average prices of these raw materials in 2021 were PLN 70/t for the sand and PLN 105/t for the gravel. It was assumed that both the size grades were produced in equal proportions. The possible profit was considered in terms of various coalitions, including mining in a single district, in an alliance of two districts and a grand coalition composed of three local governments. The latter arrangement ensures the production of 300,000 tons of raw material and was adopted based on the amounts reported in previous years when the mining activities were performed in the territory of all districts. Shares in the aggregate production of individual districts were derived from the size of the deposit area in a given district. The possible profit from the exploitation and sale of the raw material was adopted as the basis for concluding the method of dividing the mining royalty between the communes. This profit is generated by the amount of mineral extraction and its calculation is built upon the income approach, based on annual cash flows, and not on accounting profit. In this approach, the profit was assumed to be the difference between the selling price of the raw material and the operating margin, generally referring to production costs. The standard operating margin is usually assumed to be around 15%, although in the analysis, it was made dependent on the production volume wherein the higher the volume of the raw material produced, the higher the margin, and thus the higher the unit profits resulting from the scale effect reducing fixed production costs achieved through the more efficient

use of production lines. Table 2 includes the feasible annual financial (cash) flows depending on the level of mining production.

Table 2.   Hypothetical amounts of extracted minerals and profits from the Racibórz II-Reservoir 5 deposit

Tabela 2.   Hipotetyczne ilości wydobycia kopalin i zyski ze złoża Racibórz II-Zbiornik 5

Coalition	Hypothetical mining output (tons)	Gross margin (%)	Hypothetical total profit from production (PLN)	Unit profit (PLN/t)
Racibórz (R)	47,700	7	292,162,5	6.12
Kornowac (K)	69,300	8	485,100.0	7.00
Lubomia (L)	183,000	11	1,761,375.0	9.63
Racibórz and Kornowac (R,K)	117,000	9	921,375.0	7.88
Racibórz and Lubomia (R, L)	230,700	12	2,422,350.0	10.50
Kornowac and Lubomia (K, L)	252,300	13	2,869,912.5	11.38
Racibórz, Kornowac and Lubomia (R, K, L)	300,000	15	3,937,500.0	13.13

The values given in column 4 of Table 2 are the values of characteristic functions of particular coalitions. By limiting the core conditions to the minimum payoffs of one-man coalitions, solutions $x = (x_1, x_2, ..., x_n)$ were calculated for the system of inequalities:

- $x_1 \geq \upsilon(R) \geq 292,162.5$;
- $x_2 \geq \upsilon(K) \geq 485,100.0$;
- $x_3 \geq \upsilon(L) \geq 1,761,375.0$;
- $x_1 + x_2 \geq \upsilon(R,K) \geq 921,375.0$;
- $x_1 + x_3 \geq \upsilon(R,L) \geq 2,422,350.0$;
- $x_2 + x_3 \geq \upsilon(K,L) \geq 2,869,912.5$;
- $x_1 + x_2 + x_3 = \upsilon(R,K,L) = 3,937,500.0$.

Solving the system leads to the creation of the set shown in Figure 1.

Since objections and counter-objections involve only coalitions of two districts, Aumann-Maschler's stable bargaining sets will constitute the section of the core marked in Figure 1. If the grand coalition (composed of three districts) takes into account any payoff vector belonging to this set, it follows from the definition of the core that none of the districts will be able to formulate an objection, since the conditions for objections $\forall_{k \in S} y_k > x_k$ and $\Sigma_{k \in S} y_k = \upsilon(S)$ cannot be fulfilled simultaneously. The polygon *ABCDEF* is a set of non-dominated imputations that take into account the individual rationality of a single entity, the collective rationality (of all districts) and the coalition rationality for coalitions of two
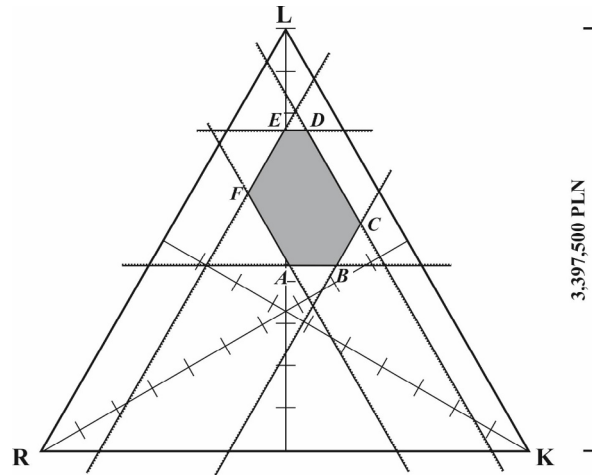
Fig. 1. Aumann-Maschler bargaining sets (grey polygon ABCDEF) for any arrangement of districts that allowed for the extraction of minerals from the Racibórz II-Reservoir 5 deposit

Rys. 1. Zestawy przetargowe Aumanna-Maschlera (szary wielokąt ABCDEF) dla dowolnego układu okręgów, które pozwoliły na wydobycie kopalin ze złoża Racibórz II-Zbiornik 5

districts. For any chosen solution from the set, members of all two-party coalitions receive a total payoff that, at least, is as much as this alliance can secure for itself. The Aumann-Maschler set is a closed convex set, and each imputation belonging to it is stable in the sense that no coalition would simultaneously want to change the result of the division or be able to do so. The bargaining set does not have to be a single vector and what is more, the division proposals belonging to the stable solution do not have to be "fair". For example, district Racibórz would rather not be satisfied with the division proposed in point "C" $x = (x_R = 292,162.5; x_K = 151,515.0; x_L = 2,130,187.5)$; however, the district could not indicate any coalition in which itself, as well as the district allying with the district Racibórz, would receive more. The possible threat of quitting the coalition is not considered because it has been assumed, following Aumann-Maschler's theory, that the rational management of the resources of Racibórz II-Reservoir 5 deposit, requires the involvement of all districts in the mining activities. Values of vectors at the vertices of the convex polygon *ABCDEF* representing the Aumann-Maschler set are presented in Table 3.

Based on the solution of the game of potential revenues from the exploitation of the Racibórz II-Reservoir 5 deposit, the obtained results were transformed into the allocation of revenues from the mining royalty between the districts. The data in Table 1 shows that mining is currently performed in two districts. Exploitation in Kornowac district has been discontinued. Would any payoff vector from the Aumann-Maschler set be a significant incentive for Kornowac authorities to resume mining? The minimum values resulting from the percentage shares in payoffs were adopted in the considerations in the first place. It is worth mentioning that the royalty paid in advance to the district would be a rate reflecting its

Table 3.   Coordinates of the polygon vertices representing the extreme possible imputations of the payoff vector for the districts

Tabela 3.   Współrzędne wierzchołków wielokąta reprezentujące skrajne możliwe imputacje wektora wypłat dla okręgów

Polygon vertices	Racibórz	Kornowac	Lubomia	Percentage of the payoff		
				Racibórz	Kornowac	Lubomia
A	1,067,587.5	1,108,537.5	1,761,375.0	27.11	28.15	44.73
B	660,975.0	1,515,150.0	1,761,375.0	16.79	38.48	44.73
C	292,162.5	1,515,150.0	2,130,187.5	7.42	38.48	54.10
D	292,162.5	629,212.5	3,016,125.0	7.42	15.98	76.60
E	436,275.00	485,100.0	3,016,125.0	11.08	12.32	76.60
F	1,067,587.5	485,100.0	2,384,812.5	27.11	12.32	60.57
Average	549,832.5	1,050,630.0	2,337,037.5	13.96	26.68	59.35

readiness to sanction exploitation on its own territory. Table 4 includes the relevant chapters assuming a minimum share of Kornowac commune at a level of 12.32%.

The districts where the exploitation is performed receive 87.68% of the expected mining royalty revenues. The royalty for Kornowac district is an incentive and at the same time an obligation for Kornowac local authorities to consent to the exploitation. This payoff can also be understood as a kind of compensation resulting from the cross-border (between the district) impact of mining on the environment, despite the lack of direct mining works in the Kornowac district area. The districts that do not consent to mining in their area at a given moment and remain the beneficiaries of the mining royalty declare their

Table 4.   The division of the mining royalty for the minimum value of payoffs for Kornowac district according to the Aumann-Maschler solution

Tabela 4.   Podział opłaty eksploatacyjnej dla minimalnej wartości odpłat dla rejonu Kornowac według rozwiązania Aumanna-Maschlera

	Base division	New division	Base division	New division
	2020		2021	
Racibórz	19,625	17,207	13,210	11,583
Kornowac	0	14,350	0	14,230
Lubomia	96,855	84,923	102,290	89,687
Total	116,480		115,500	

acceptance of mining and readiness in the near future, e.g. in a five-year perspective. If the district fails to comply with the arrangements regarding the commencement of exploitation (withdrawal of consent, change of local plans, resignation from mining by the entity using the deposit, etc.), it would be necessary to return all or part of the revenues received from the exploitation royalty by the arrangements and regulations. Determining the length of the period of payment of the levy to the district where mining is not performed could depend on the size of the deposit resources within the district area or the anticipated duration of mining.

Below, hypothetical variants of stopping mining in one or two of the districts are considered and given in Table 5, adopting the percentage share determined based on the peaks of the Aumann-Maschler bargaining set as a criterion for allocating part of the mining royalty at the minimum, medium and maximum level. Hypothetical production volumes from Table 2 and the royalty rate for 2021 (PLN 0.66/t) were adopted for the calculation. Adoption of the maximum share for any of the districts seems to be unfair and "unjust", despite the availability in the set, thus the acceptance of the minimum values seems to be acceptable. The rates vary concerning the distribution of deposit resources in the territory of each district and the highest rates are assigned to Lubomia district. It is reasonable that the largest shareholder in terms of the volume of the resources should receive the greatest incentive to join the coalition exploiting the deposit.

Under geological and mining law in Poland, when a mine operates in more than one district, one county or one voivodship, the royalty is calculated in proportion to the size of the area covered by the activity or the amount of mineral extracted from a given area. An interesting aspect of the objective division of the royalty between districts may be the use of the central core point, which also belongs to the Aumann-Maschler set (Einy and Wettstein 1996). This point corresponds to the vector of payoffs of the bargaining set, meeting the conditions:

$$x = (921,375.0 - x_R - x_K = 2,422,350.0 - x_R - x_L = 2,869,912.5 - x_K - x_L)$$

with the total payoff of the grand coalition $x_R + x_K + x_L = 3,937,500.0$. The solution to the equality is imputation $x = (x_R = 513,800.0; x_K = 961,362.5; x_L = 2,462,337.5)$, which defines the division of the mining royalty in the proportions of 13.05% (Racibórz), 24.42% (Kornowac) and 62.54% (Lubomia).

In the case study of the Racibórz II-Reservoir 5 deposit, the Kornowac district portion of the deposit is not exploited. From the point of view of all interested parties, it seems acceptable to divide the mining royalty in such a way that it takes into account the minimum values of the Aumann-Maschler solution or corresponds to the central point of the core, also belonging to the bargaining set (Table 6). A practical interpretation of the division of royalties would require payments to the non-mining district in the amount resulting from the minimum Aumann-Maschler solution, and after joining the coalition, the payoff would be made in the amount corresponding to the central point of the core.

Table 5.   Hypothetical divisions of the mining royalty in the districts for two-party and single-party coalitions
            based on the solutions of the Aumann-Maschler bargaining set
            (the districts where mining is not continued are in bold)

Tabela 5.   Hipotetyczny podział władzy górniczej w okręgach koalicji dwupartyjnej i jednopartyjnej
            w oparciu o rozwiązania układu przetargowego Aumanna-Maschlera
            (okręgi, w których nie jest kontynuowane wydobycie, zaznaczono pogrubioną czcionką)

District	Mining output (tonnes)	Total royalty (PLN)	Royalty payable including Aumann-Maschler solution		
			Min.	Avg.	Max.
Racibórz	**0.00**	**0.00**	**12,355.64**	**23,245.91**	**45,143.03**
Kornowac	69,300.00	45,738.00	42,344.24	39,352.98	33,338.43
Lubomia	183,000.00	120,780.00	111,818.12	103,919.11	88,036.54
Total	252,300.00	166,518.00	166,518.00	166,518.00	166,518.00
Racibórz	47,700.00	31,482.00	27,603.42	23,082.60	19,367.73
Kornowac	**0.00**	**0.00**	**18,758.68**	**40,623.50**	**58,590.42**
Lubomia	183,000.00	120,780.00	105,899.90	88,555.90	74,303.86
Total	230,700.00	152,262.00	152,262.00	152,262.00	152,262.00
Racibórz	47,700.00	31,482.00	17,400.10	12,797.43	7,366.79
Kornowac	69,300.00	45,738.00	25,279.39	18,592.50	10,702.69
Lubomia	**0.00**	**0.00**	**34,540.51**	**45,830.07**	**59,150.52**
Total	117,000.00	77,220.00	77,220.00	77,220.00	77,220.00
			Min.	Avg.	
Racibórz	**0.00**	**0.00**	**8,961.88**	**16,860.89**	
Kornowac	**0.00**	**0.00**	**14,880.10**	**32,224.10**	
Lubomia	183,000.00	120,780.00	96,938.03	71,695.01	
Total	183,000.00	120,780.00	120,780.00	120,780.00	
Racibórz	**0.00**	**0.00**	**3,393.76**	**6,385.02**	
Kornowac	69,300.00	45,738.00	21,885.63	12,207.47	
Lubomia	**0.00**	**0.00**	**20,458.61**	**27,145.50**	
Total	69,300.00	45,738.00	45,738.00	45,738.00	
Racibórz	47,700.00	31,482.00	13,521.52	4,398.04	
Kornowac	**0.00**	**0.00**	**3,878.58**	**8,399.40**	
Lubomia	**0.00**	**0.00**	**14,081.90**	**18,684.57**	
Total	47,700.00	31,482.00	31,482.00	31,482.00	

Table 6.   Proposed divisions of the mining royalty based on Aumann-Maschler's bargaining sets

Tabela 6.   Proponowane podziały honorarium górniczego na podstawie układów przetargowych Aumanna-Maschlera

District	Base solution	Minimum value of the Aumann--Maschler set	Core central point	Base solution	Minimum value of the Aumann--Maschler set	Core central point
	2020			2021		
Racibórz	19,625	17,207	15,201	13,210	11,583	15,073
Kornowac	0	14,350	28,433	0	14,230	28,194
Lubomia	96,855	84,923	72,846	102,290	89,687	72,233
Total	116,480			115,500		

Conclusions

According to the World Bank nomenclature, royalties refer to taxes and levies, the purpose of which is to compensate the owner of irretrievably lost mineral deposits or they are specific to the mining sector (Otto 2001; Otto et al. 2006). A *royalty* is thus a kind of payment for the use of the natural environment. The mining royalties in Poland are generally divided between the state and local governments. The share of the *royalty* is defined in proportions stipulated by law, although in other countries, the royalty rates are dictated by temporal volatility in relation to the current budgetary situation, and are often strongly dependent on political, social and economic factors. It seems, however, that the dominant function of the royalty is fiscal, aimed at supplying the authorities (central and local) with financial resources that are necessary to perform their tasks, and its incentive effect in the context of resource conservation is suppressed.

The size of the potential own revenue of the district, resulting from a specific use of land, can be a factor encouraging or discouraging one or another activity (Nikitina 2014). The allocation of land for mining activities is one of the many possibilities of land use by the district authorities and may determine the benefits derived from it. In Poland, the highest revenues of districts are usually generated by real estate taxes, local taxes and levies established and collected by local government units on the basis of separate acts (Sekuła 2010; Kaźmierczak 2015; Śleszyński et al. 2021). Comparing the potential revenues from mining and agricultural activities, the levies to the local government from mining are many times higher than those generated by farm taxes. A significant increase in revenues from mining royalties in relation to the agricultural tax is undoubtedly a factor encouraging consent to mining.

This paper has proposed a non-traditional approach to the division of the mining royalty. The proposal is a combination of a payment mechanism for a unit of extracted mineral with an income mechanism. With regard to revenues, the royalty does not deplete it by itself but is a kind of derivative of this marriage. The indicated royalty division involves all communes within the area where the mineral deposit is located. Own revenues from exploitation are divided fairly in accordance with the bargaining set solution. Consideration of the communes, where no exploitation is performed in the royalty payments is a significant extension of the royalty functionality. A commune where no mining operations are undertaken is burdened with negative external effects and any, even nominal fee compensation for adjacent mining activity seems appropriate. Separating this part of the royalty for the commune is possible, as demonstrated, by a solution based on Aumann-Maschler bargaining set. Any consent to exploitation in one's own area would result in further, additional (greater) income from the mining royalty. Not without significance is the fact that such a compromise combines the desire of the local government to obtain funds from mineral exploitation to finance its own tasks from one hand to the deposit protection requirement and full use of mineral resources from the second. Additional incentives for the local government through the proposed division of the mining royalties, taking into account that no mining activities are performed in the district having resources, appears to be a right and justified action, despite some arbitrariness. The solution seems fair from the point of view of game theory and meets the criteria of coalition equilibrium. In technical, formal and legal terms, this solution would probably require more details and legal normalization to regulate the principles of collection of mining royalties, withdrawal and/or accession of municipalities to mining coalitions.

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THE AUMANN-MASCHLER BARGAINING SET AS A CRITERION FOR THE DIVISION OF MINING ROYALTY

Abstract

This article proposes an unconventional approach to the division of mining royalties between various administrative units affected by mining activities. Typically, a mining royalty is considered the own income of the units where the exploitation is performed and is usually calculated in relation to the tonnage (volume) of the extracted mineral or the value of the raw material produced. In the article, a different approach has been proposed in order to ensure a useful and fair division of the royalty wherein the unit levy calculation approach was combined with the income criterion using Aumann-Maschler bargaining sets. The case study of the Racibórz II-Reservoir 5 pebble deposit, located within three administrative units (districts) in southern Poland was considered. The exploitation of the deposit within each of the districts requires the separate consent of the local authorities, and in the analyzed case, it is currently conducted in two districts. In terms of income, achievable revenues from exploitation for various alliances of the districts that provide the deposit for mining were calculated. The feasible revenues were transformed into appropriate streams of the mining royalty distribution. It was pointed out that the solutions suggested by the Aumann-Maschler bargaining sets can be treated as a fair division. Proper royalty allocation can be an effective, flexible and important factor in encouraging the district authorities to consent to the exploitation. The adoption of solutions based on the Aumann-Maschler bargaining set will meet the requirement of the full use of mineral resources and is an example of externalities compensation resulting from mining activities.

ZBIÓR PRZETARGOWY AUMANNA-MASCHLERA JAKO KRYTERIUM PODZIAŁU OPŁATY EKSPLOATACYJNEJ

Streszczenie

W artykule zaproponowano niekonwencjonalne podejście do podziału opłaty eksploatacyjnej pomiędzy różne jednostki administracyjne objęte działalnością wydobywczą. Zazwyczaj opłata eksploatacyjna jest dochodem własnym jednostek, gdzie prowadzona jest eksploatacja i wylicza się ją najczęściej w odniesieniu do tonażu (objętości) wydobytej kopaliny lub wartości wyprodukowanego surowca. W artykule, poszukując użytecznego i sprawiedliwego podziału tej daniny, połączono podejście naliczania opłaty jednostkowej z kryterium przychodowym, wykorzystując zbiory przetargowe Aumanna-Maschlera. Rozważono studium przypadku złoża kruszyw Racibórz II-zbiornik 5,

położonego w południowej Polsce w obrębie trzech gmin. Eksploatacja złoża w obrębie każdej z gmin wymaga odrębnej zgody samorządu lokalnego, a w analizowanym przypadku prowadzona jest obecnie na terenie dwóch z nich. Skalkulowano, w ujęciu dochodowym, możliwe do osiągnięcia przychody z tytułu eksploatacji dla różnorodnych aliansów gmin aprobujących wydobycie. Możliwe do osiągnięcia przychody przekształcono w odpowiednie strumienie rozdziału opłaty eksploatacyjnej. Wskazano, że rozwiązania sugerowane zbiorami przetargowymi Aumanna-Maschlera umożliwiają elastyczne zarządzanie dystrybucją opłaty, a właściwa jej alokacja może być skutecznym i istotnym czynnikiem aktywizującym gminy do wyrażenia zgody na eksploatację. Zademonstrowano, że przyjęcie rozwiązań na bazie zbioru przetargowego Aumanna-Maschlera spełnia postulat racjonalizacji gospodarki złożem, umożliwiając kompletne i kompleksowe wykorzystanie zasobów, zapewniając przy okazji, podyktowany kryteriami rozwiązania, sprawiedliwy podział opłaty, jako kompensaty efektów zewnętrznych wynikających z wydobycia i całościowej działalności górniczej.