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A priori determining of the accuracy of mineral resources volume determination

Introduction – methods of measuring surfaces for the purpose of calculating volumes

The methods of measuring surfaces for the purpose of the subsequent determination of the volume of excavated or layered rock are as follows.

1. Methods based on the estimation and experience of the mining surveyor – only for minor open cast mine mines.
2. Methods based on the summation of units (weight or volume) transported from the quarry area.
3. Methods based on the measurement of the rock volume “on the conveyor belt”, usually using the profile scanning and determining the speed of the belt movement or by determining the weight of the rock on the conveyor belt.
4. Methods based on the geodetic measurements of the discrete points and shapes of the excavated material and comparing the subsequently created digital model with the model of the previous phase of the measurement.

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5. Methods based on terrestrial laser scanning using static lasers and mobile lasers while driving or using the Stop and Go method and airborne laser scanning methods, whether from a manned aircraft or remotely piloted aircraft systems.
6. Methods based on the use of aerial photogrammetry based either on measuring the digital surface model through stereoscopic observation with expression of the digital surface model of discrete points and field edges (obligatory connecting lines), or by methods of correlation calculations resulting in a dense cloud of points forming a digital surface model both from the images acquired by aircraft with crew or remotely piloted air systems.

1. Methods

1.1. Methods of calculating volumes

Methods of measuring the terrain surface in an open cast mine aiming at determining the volume, usually including the area of the mine workings between the two measurement phases are generally carried out in accordance with the further processing of this data and calculating the volume based on the difference of the two measured digital surface models. Volumes are now mostly determined by comparing two digital surface models using special software. Most computational methods for determining the rock volume for the purpose of the theoretical determination of the volume accuracy are listed below. In determining the volume, the following procedures are generally used:

- ◆ dividing the object into elementary geometric spatial parts and calculating the volumes of these parts,
- ◆ dividing the objects by parallel sections, the volume calculation based on the average of the areas of two neighboring sections, and knowledge of the distance between the sections. Formally, this method is divided into the calculation from horizontal sections, i.e. contours, and the calculation of vertical sections, i.e. profiles,
- ◆ dividing the solid limited by two digital surface models by the system of mutually perpendicular planes with regular distances and calculating the volume as the sum of all individual “replacement” four-sided prisms,
- ◆ dividing the solid limited by two digital surface models into triangular prisms when three prism edges are in the direction of one of the axes, and subsequently the sum of volumes of all of these triangular prisms,
- ◆ dividing the solid into tetrahedrons, calculating their volumes, and the volume calculation as the sum of all tetrahedrons formed by connecting the tops of two digital surface models

If the digital model consists of discrete points (or obligatory connecting lines – terrain edges), the only accurate volume calculation of the above division of solids and the subse-

quent calculation of volume is that under point e). This calculation is significantly more time consuming than other methods of solid division and the subsequent calculation of volume, but it is the only one that does not have to solve averaging heights and areas and the additional calculation of certain parts of the body by rough calculations or their replacement by approximate spatial objects.

If the digital model is formed and described as a difference or intersection of at least two of the accurate spatial surfaces (planes, cylinder portions, cones, paraboloids, spheres, and other clearly definable spatial surfaces), the volume calculation is only a question of optimization of the calculation of the surfaces formed of these spatial objects concluding and defining the calculated volume.

1.2. Comparing methods of calculating volumes

The issue of calculating volumes is described and processed e.g. in (Michalčák et al. 1990). The method of determining the volume using the layers was completely processed by VÚGTK, v.v.i. (Research Institute of Geodesy, Topography and Cartography, public research institution) between 1984 and 1988. The research reports (Šanda 1989) are based on the fact that the volume should be determined with an accuracy of 0.33% of the total volume, but this is the case of the absolutely definite surface of tanks for oil products, so stating the need to determine the volume with an accuracy of 1%, more closely justified in the last paragraph of this chapter, is acceptable, but relatively borderline in volumes of hundreds of thousands of cubic meters. Specific applications and solutions for the given cases are mentioned e.g. by (Černota et al. 2012; Mučková et al. 2012). The general concept of the theoretical determination of the volume of the exact solid and the conclusions for the present photogrammetric mining practice have not been published in the current literature yet. In VÚGTK, v.v.i., this issue was studied between 1964 and 1969 under the leadership, Professor Jiří Šíma, CSc, Eng. Practical methods for the technology of determining the rock volume in open cast mining were introduced. To carry out an, a priori analysis of the accuracy of determining the volume of the excavated rock, consider a precisely defined solid, namely a cylinder segment 600 m long with a 6 m long front-end and a 70 m long cylinder chord with the total volume of 168.983 m³. Thus defined exact solid (see Fig. 1) and its clearly defined initial capacity will be further analyzed in the text using the above-mentioned options for calculating the volume of the solid. The solid is identical to the shape of the homogenization coal stock of the Nástup Tušimice Mine, and it can be used to set out the facts concerning the needs and requirements for photogrammetric measurement methods.

In Figure 1, Euclid's theorem applies for the triangle **TUV** constructed at the forefront of the cylinder (with the cylinder axis $|CC'|$), when knowing the value of **h** (the line segment **PT** – the average height of coal in the middle of the coal stock) and the chord length **S** (the line segment **UX** – coal stock width 70 m) the following applies:

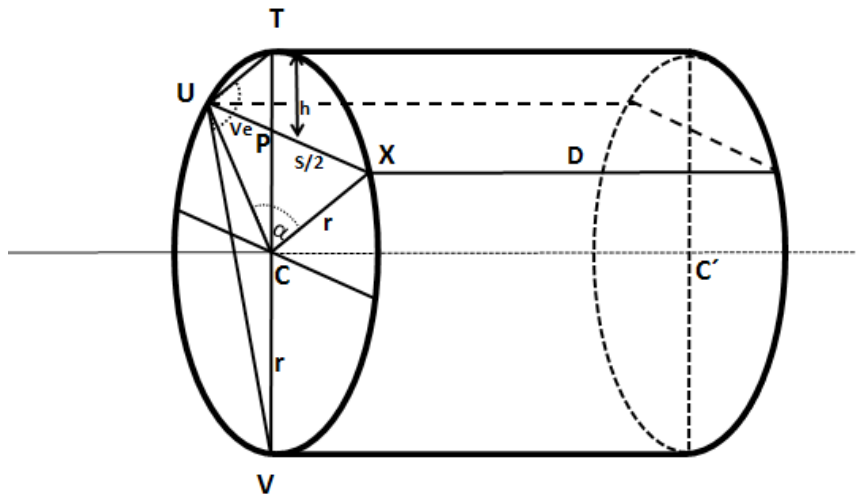


Fig. 1. Graphical representation of the cylindrical segment

Rys. 1. Przedstawienie graficzne segmentu cylindrycznego

$$|UP| \cdot |UP| = |PT| \cdot |PV| \quad (1)$$

the unknown segment $|PV|$ can be calculated from that as

$$|PV| = (|UP| \cdot |UP|) / (|PT|) \quad (2)$$

and it applies that

$$|PT| + |PV| = d = 2r \quad (3)$$

After receiving the calculated r based on the knowledge of the chord length S (the input value – twice the height V_e in the Euclidean triangle), the angle in radians can be calculated as

$$\alpha = 2 \arcsin (S/2r) \quad (4)$$

and then through calculating the difference in the area of the entire sector of a circle CU_CX (according to the formula for the area of a sector of a circle $A_v = (\alpha \cdot r \cdot r)/2$) from the area of the triangle UXC (with an area $A_t = r \cdot r \cdot (\sin(\alpha/2)) \cdot (\cos(\alpha/2))$), and we obtain the final area of the sector of a circle A . The value of cylindrical segment volume V is only the product of $A \cdot D$.

The final calculation of the exact volume of the cylindrical segment of the typical shape of the homogenization coal stock of the Nástup Tušimice Mine is shown in Table 1.

Table 1. Listing of fundamental values of the cylindrical segment volume

Tabela 1. Zestawienie podstawowych wartości objętości segmentu cylindrycznego

Enter the height of the rotating cylinder segment h (the line segment TP) in (m)	6	The calculated area of the sector of a circle associated with angle α in (m ²)	3.750
Enter the chord length of the rotary cylinder S (i.e. line segment UX) in (m)	70	The calculated area A of the front of the cylindrical segment in (m ²)	282
Enter the length of the rotating cylinder D in (m)	600	The calculated volume V of the cylindrical segment in (m ³)	168.983

In further tests with respect to calculating the volume of the known mathematical solid, the admissible error in determining the volume is considered the possible, or permissible error in determining the volume of up to 1% of the total value, thus in round numbers, the value of 1.690 m³. This value was not chosen randomly, because in mining practice, it is usually considered the threshold value of the error in determining the volume of extracted or piled (landfilled) material in opencast mining, which is acceptable to governing bodies, for example ČBÚ (Czech Mining Office).

1.3. The method of calculating volumes using layers parallel with the segment chord

The basic volume of the exact solid was determined as described above. Calculation variants are compared below using three to twelve layers against the exact calculated value of the cylindrical segment volume. Figure 2 shows the division of the volume into six layers. In Table 2, values of calculation variants are tabulated from the value of the volume of a triangular prism with the base of the triangle **UXT** over three to twelve layers. Graphical representation of the ratio of the number of sections to the value of the resulting error in determining the volume (Graph 1) is also based on Table 2. The volume was calculated as the sum of volumes of the **V_i**-th layer, which is equal to the sum of the areas of this layer and the area adjacent to this area multiplied by half of the height value (distantness) of these adjacent layers.

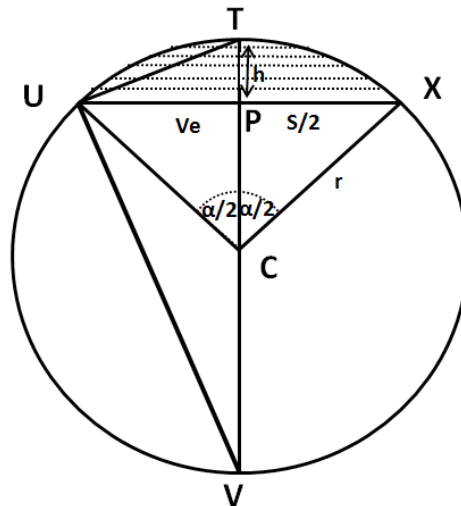


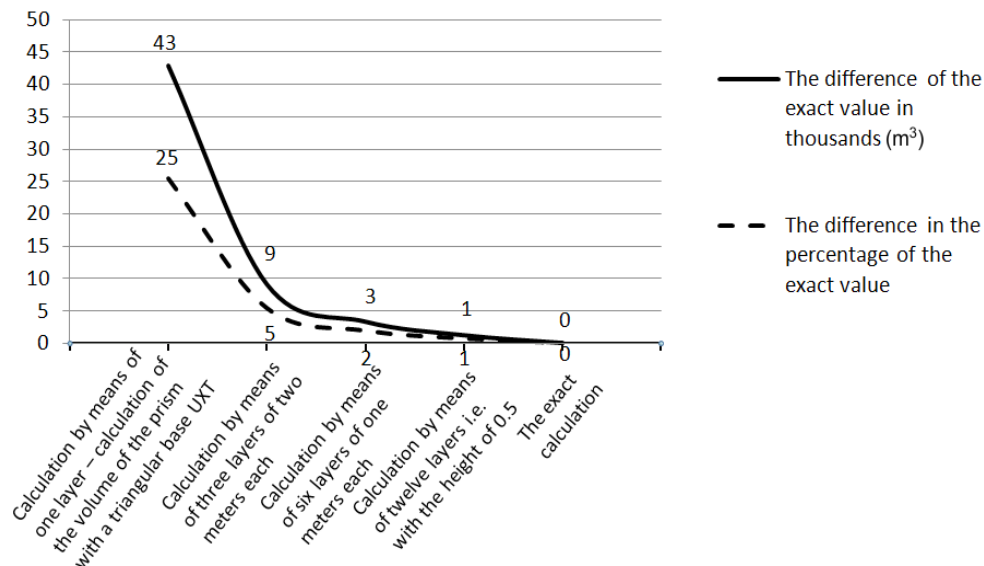
Fig. 2. Graphical representation of calculating the volume using layers; the figure for the variant with six layers

Rys. 2. Przedstawienie graficzne obliczania objętości za pomocą warstw;
rysunek dla wariantu z sześcioma warstwami

Table 2. Variants of calculating the volume of the cylindrical solid segment by means of layers

Tabela 2. Warianty obliczania objętości cylindrycznego segmentu bryłowego za pomocą warstw

Volumes in layers	The solid volume in thousands (m ³)	The difference from the exact value in thousands (m ³)	The difference from the exact value (%)
Calculation by means of one layer – the prism volume calculation	126	42.9	25.1
Calculation by means of three layers of two meters each	160	9.1	5.4
Calculation by means of six layers of one meter each	166	3.3	1.9
Calculation by means of twelve layers i.e. with the height of 0.5 meters each	168	1.2	0.7
The exact calculation	169	0	0



Graph 1. Comparison of the volume determination according to the number of layers used to calculate the volume

Wykres 1. Porównanie oznaczania objętości w zależności od liczby warstw użytych do obliczenia objętości

1.4. The Method of Calculating Volumes by Means of Profiles

For the calculation of the value by means of profiles, it is necessary to calculate the surface area of the front profile, which is equal to the sum of areas n of isosceles triangles with arms r and the vertex angle α/n , where the surface area of one isosceles triangle is calculated as

$$Ar = ((r \cdot r)/2) \cdot \sin((\alpha/n)/2) \quad (5)$$

wherein the values of r and α are known from the previous paragraphs.

The sum of all surface areas n of the triangles is subtracted from the surface area of the triangle UXC . The resulting surface profile is multiplied by the coal stock length D to calculate the volume. Figure 3 illustrates the calculation of the surface area of the profile for the hexagon segment. The corresponding values are tabulated in Table 3. Graph 2 shows the dependence of the number of triangles on the quality of the determination of the cylindrical segment volume by means of calculating the profiles and alternatives of n -edge segments.

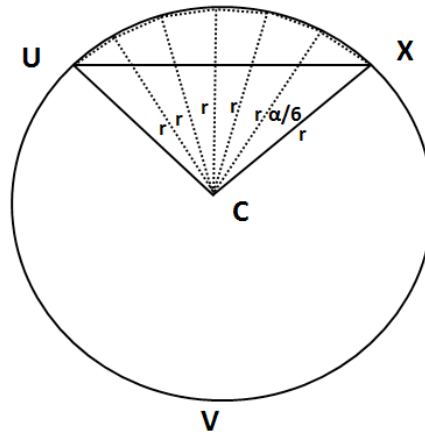


Fig. 3. Graphical representation of the volume calculation for the variant with the hexagonal segment

Rys. 3. Przedstawienie graficzne obliczania objętości dla wariantu z segmentem heksagonalnym

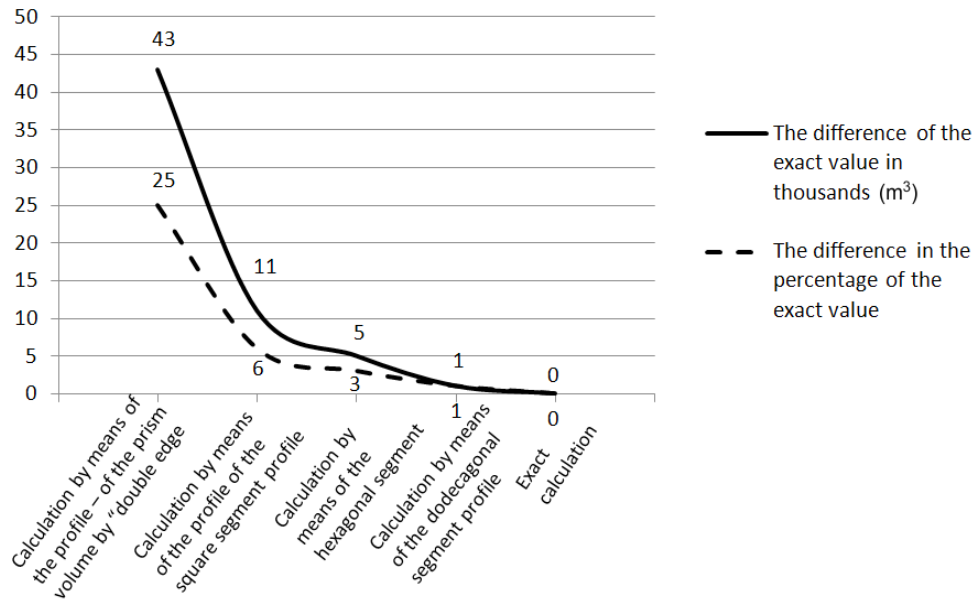
Table 3. Variants of calculating the volume of the cylindrical segment solid by means of profiles

Tabela 3. Warianty obliczania objętości cylindrycznego segmentu bryłowego za pomocą profili

Volumes in profiles	The solid volume in thousands (m ³)	The difference from the exact value in thousands (m ³)	The difference from the exact value (%)
Calculation by means of the profile – of the prism volume by “double edge segment”	126	42.9	25.4
Calculation by means of the profile of the square segment profile	158	10.8	6.4
Calculation by means of the hexagonal segment profile	164	4.8	2.8
Calculation by means of the dodecagonal segment of the profile	168	1.2	0.7
Exact calculation	169	0.0	0.0

1.5. The Calculation Method by Determining the Mean Height to the Solid Surface Area in the Regular Network

The base of the cylindrical segment $UXX''U''$ shown in Figure 4 is in the horizontal plane. The base is divided into a regular sized grid of the value Kg . Perpendiculars raised



Graph 2. Comparison of determining volumes by the number of vertices on the profile segment

Wykres 2. Porównanie oznaczenia objętości za pomocą liczby wierzchołków w segmencie profilu

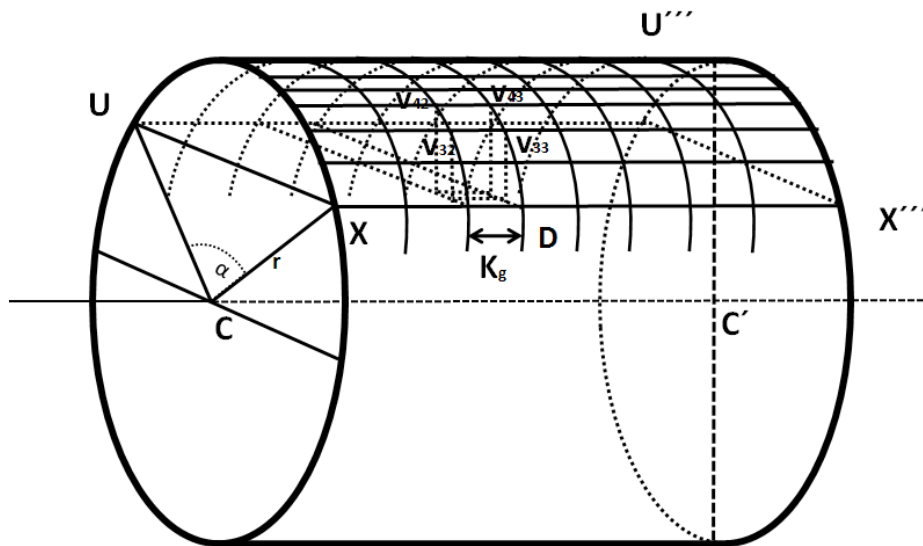


Fig. 4. Graphical representation of calculating the volume by specifying values for the volume of replacement prisms

Rys. 4. Przedstawienie graficzne obliczania objętości poprzez określenie wartości objętości pryzm zastępczych

in the nodes of the grid are directed to the surface of the cylindrical segment and intersect the segment surface at points V_{32} , V_{33} , V_{42} and V_{43} , as shown in Figures 4 and 5. For the selected step K_g , heights of the perpendiculars above the base at nodal points of the grid were calculated as illustrated in Figure 6. The volumes of individual replacement prisms are calculated by multiplying

$$K_g \cdot K_g \cdot ((\text{sum } (V_{ij}, V_{i,j+1}, V_{i+1,j} \text{ a } V_{i+1,j+1})/4) \quad (6)$$

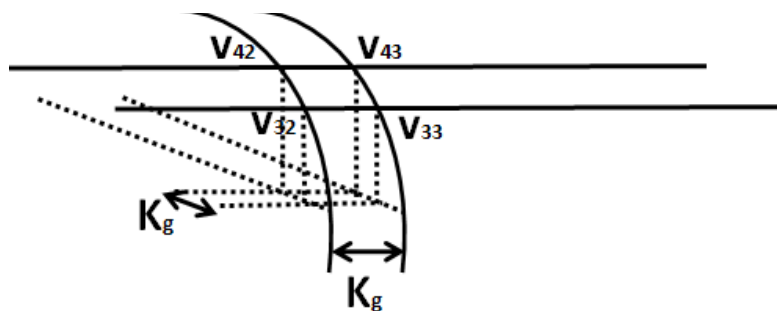


Fig. 5. A detailed illustration of the replacement prism

Rys. 5. Szczegółowa ilustracja pryzmy zastępczej

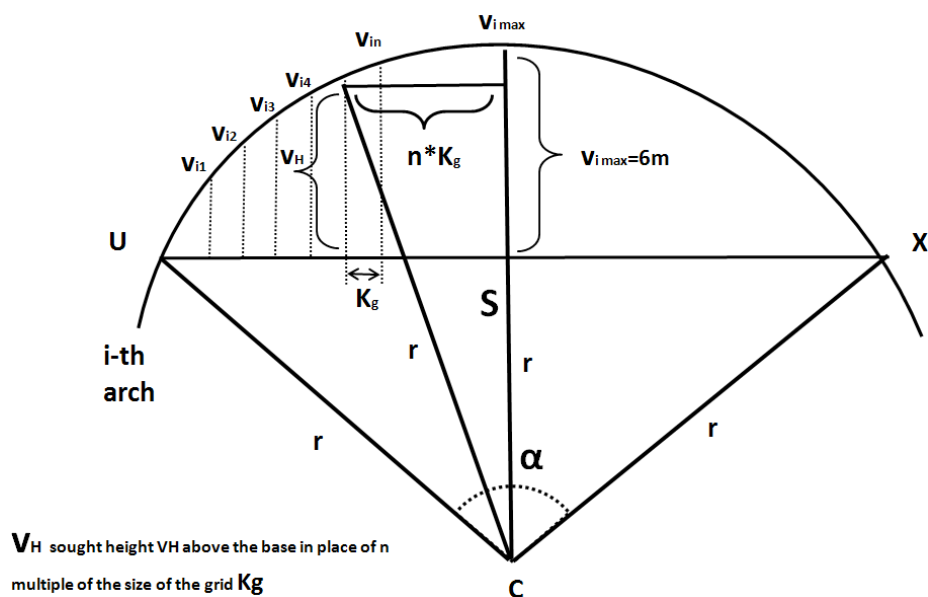


Fig. 6. A detailed illustration of the determination of heights in the replacement prism

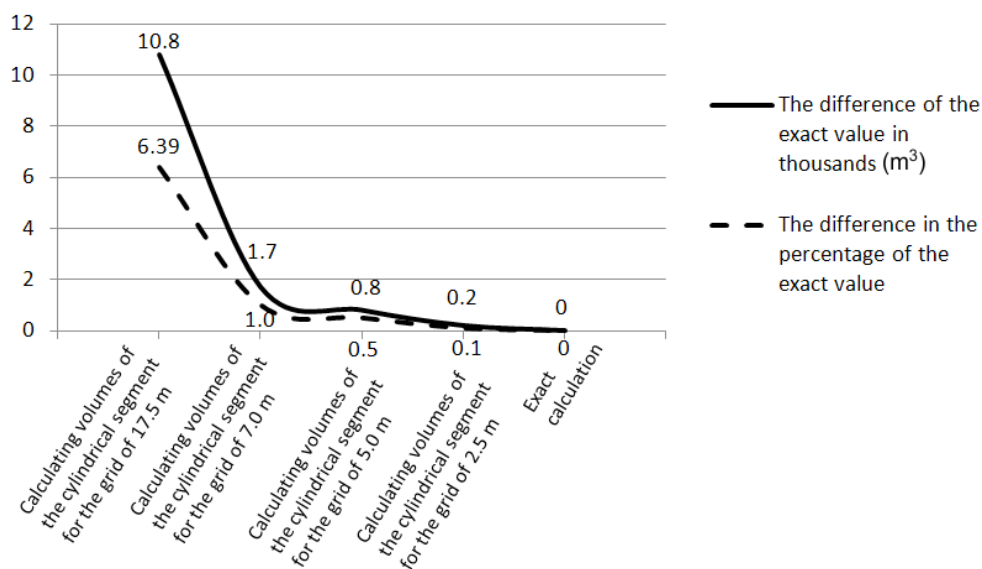
Rys. 6. Szczegółowa ilustracja określenia wysokości w pryzmie zastępczej

Volumes were calculated for the step of the grid **Kg**: 2.5 m, 5 m, 7 m and 12.5 meters. Table 4 summarizes the volume values, and Graph 3 graphically illustrates the tabulated values.

Table 4. Variations of calculating the solid volume by means of a regular grid **Kg** from 2.5 to 17.5 m

Tabela 4. Warianty obliczania objętości stałej za pomocą zwykłej siatki **Kg** od 2,5 do 17,5 m

Volumes determined from the volume of replacement prisms	The solid volume in thousands (m ³)	The difference from the exact value in thousands (m ³)	The difference from the exact value (%)
Calculating volumes of the cylindrical segment for the grid of 17.5 m	158	10.8	6.39
Calculating volumes of the cylindrical segment for the grid of 7.0 m	167	1.7	1.0
Calculating volumes of the cylindrical segment for the grid of 5.0 m	168	0.8	0.5
Calculating volumes of the cylindrical segment for the grid of 2.5 m	169	0.2	0.1
Exact calculation	169	0.0	0.0



Graph 3. Comparison of determining volumes by the grid step size **Kg**

Wykres 3. Porównanie oznaczania objętości za pomocą kroku siatki **Kg**

2. Results and discussion

2.1. Dependence of the deviation value of the input height measurement on the volume calculation

Finding out the dependence of the influence of height deviations of the measured points entering the final volume calculation was done using the volume calculation by determining the value of the mean height to the surface area of the solid in the regular grid (i.e. according to paragraph 3.4). Volume values were calculated when the heights of all points on the surface of the cylindrical segment were loaded by systematic error. The calculation of this systematic error was made when its value was +3.5 cm, +5.0 cm, +7.5 cm, and +10 cm. In these alternatives of the values of systematic errors added to the exact values, volumes for all values of the step Kg were calculated. The table shows that when measured with a systematic error of 3.5 cm and 5 cm, even the percentage volume value in relation to the exact volume “is improving”, and the volumes are determined as if they were more approaching the exact volume, which is, however, inadequate only at first glance, because in case of a concave shape of the solid, averaging four heights specified for the given prism is always an average with higher weight of lower peak heights determined on the solid (with convex surfaces – for example, when storing bulk materials along the walls of U-shaped tanks, it would be the other way round). A summary table of results is given as Table 5.

2.2. Comparing the methods of calculating volumes for the cylindrical segment solid

This article summarizes the performance of four types of calculations of the volume of the solid, which is approximately of the same shape as the homogenization coal stock of the Severočeské doly a. s. (the North Bohemian Mines, joint-stock company), the Nástup Tušimice Mine. They gradually included the following calculations:

- ◆ the exact calculation of the volume of the cylindrical segment,
- ◆ the calculation of the volume of the cylindrical segment solid by means of layers (four alternatives),
- ◆ the calculation of the volume of the cylindrical segment solid by means of profiles (four alternatives),
- ◆ the calculation of the volume of the cylindrical segment solid by specifying values for the volume of replacement prisms (four alternatives).

When comparing the above-mentioned methods to calculate volumes, the results of alternative calculations show that acceptable errors in determining volumes, namely below 1% compared to an exactly determined volume, occur in the calculation by means of 12 layers, i.e. formally with 24 replacement points around the circumference of the front

Table 5. Results of volumes under various systematic errors in the determination of the solid heights
 Tabela 5. Wyniki objętości przy różnych systematycznych błędach w określeniu wysokości bryły

Table for calculating the volumes of the cylindrical segment for the grid Kg systematic deviation value (m)	0.000			0.035			0.050			0.075		
	Total volume in (m ³)	The difference from the exact value in (m ³)	The difference in percentage	Total volume in (m ³)	The difference from the exact value in (m ³)	The difference in percentage	Total volume in (m ³)	The difference from the exact value in (m ³)	The difference in percentage	Total volume in (m ³)	The difference from the exact value in (m ³)	The difference in percentage
The exact value of the cylindrical segment volume for testing is 168,983 (m ³), the value of one percent is 1,689.8 (m ³)	158.184	-10,799	-6.4	159.654	-9,329	-5.5	160.284	-8.699	-5.1	161.334	-7.649	-4.5
Table for calculating the cylindrical segment volume for the grid Kg in 17.5 m (137 points on the cylindrical segment surface)	167.253	-1,730	-1.0	168.723	-260	-0.2	169.353	370	0.2	170.403	1.420	0.8
Table for calculating the cylindrical segment volume for the grid Kg in 5 m (1.680 points on the cylindrical segment surface)	168.100	-883	-0.5	169.570	587	0.3	170.200	1.217	0.7	171.250	2.267	1.3
Table for calculating the cylindrical segment volume for the grid Kg in 2.5 m (6.720 points on the cylindrical segment surface)	168.763	-221	-0.1	170.233	1,249	0.7	170.863	1.879	1.1	171.913	2.929	1.7

of the replacement solid. In the calculation by means of profiles, the number is 12 points on the profile describing the front of the replacement solid, and in the calculation determining the values of volumes of replacement prisms, the number is formally 11 points on the front circumference of the replacement solid.

Conclusions

For practical calculation of the volume of the cylindrical segment solid (for example, the homogenization of the coal stock of the Severočeské doly a.s. – Tušimice plant – see Figs. 7 and 8), it is necessary to determine approximately 1,000 points in the pseudo-regular layout on the surface of the solid (to determine the volume of homogenization coal stock with accuracy to 1% from whole volume), with the average value of the points determined either by air or terrestrial laser scanning or stereoscopic or correlation photogrammetric methods, and their error from the real value of the height above the reference surface from which the height (though formally) is measured must not be worse than 7.5 cm. Therefore, the surface of the terrain shape similar to the cylindrical segment can practically be determined and described by means of stereoscopic measurement in approximately 90 lines perpendicular to the axis of the cylindrical segment, i.e. the homogenization coal stock, so that at least 12 points are measured along this line from one base over the top to the other base of the coal stock. The necessary measurement can be performed over the standard models using large format cameras after the end of analytical aero triangulation approximately during one work shift. Real determination of the volume and height accuracy of the input laser and photogrammetric measurements at the homogenization of coal stock were determined so that the volume determination was safely below 1% of the volume of the stored coal.



Fig. 7. Three homogenization coal stock of Severočeské doly a.s. – Tušimice on the ortophoto

Rys. 7. Trzy składy homogenizowanego węgla Severočeské doly a.s. – Tušimice na ortofoto



Fig. 8. Three homogenization coal stock of Severočeské doly a.s. – Tušimice

Rys. 8. Trzy składy homogenizowanego węgla Severočeské doly a.s. – Tušimice

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**DETERMINING A PRIORI THE ACCURACY
OF MINERAL RESOURCES VOLUME DETERMINATION****Abstract**

Several surface measurement methods for determining the volume of deep or layered stone exist. One of the key indicators of coal extraction efficiency in open cast mining is to determine the volume of excavated rock. Procedures for determining the volume have been used for many centuries. Determining the extracted volume or layered material has been a periodically recurring role of mine-surveying practice, and mine surveyors apply different methods for its determination. The incorrect determination of the rock volume may result in large economic losses of the mining enterprise. The choice of the method for determining the volume depends on the deadline by which the determined volume has to be submitted to the superior components or the mining enterprise management, as well as on the requirements for accuracy of the volume determination, and a financial limit beyond which this volume determination has to be done. Secondary conditions for determining the volumes include the level of personnel training in the individual procedures and methods of measuring and calculating volumes, the technical standards of the enterprise, the applied instrumentation, hardware and software. The article compares the values of the accurately defined mathematical solid (a cylindrical segment) to the methods of calculating the volume normally used in mining and surveying practice and programs commonly used to calculate volumes in order to determine the threshold value of the systematic deviation in input measurements to determine the volume. The mathematical model is the basis for determining the correct volumes of the extracted material. The surface of the drawn or layered material does not form a smooth surface as a mathematical model. The process of determining volume errors on the mathematical model has been verified on the real body of coal deposition. The comparison of the determination of the errors between the digital terrain model on the mathematical body and the real homogenization coal stock is presented at the Conclusion of the article.

Keywords: volume determination; coal extraction; choice of the method

OKREŚLANIE A PRIORI DOKŁADNOŚCI OBJĘTOŚCI ZASOBÓW MINERALNYCH**Streszczenie**

Jest kilka metod pomiaru powierzchni dla określenia objętości zasobów surowców mineralnych. Jednym z kluczowych wskaźników wydajności wydobywania węgla w górnictwie odkrywkowym jest określenie objętości wydobytej skały. Procedury określania objętości były stosowane przez wiele stuleci. Określanie wydobytej objętości lub warstwy surowca jest okresowo powtarzającą się praktyką geodezyjną, a geodeci stosują różne metody jej określania. Nieprawidłowe określenie objętości skały może skutkować dużymi stratami ekonomicznymi przedsiębiorstwa górniczego. Wybór metody określania objętości zależy od terminu, w którym ustalona objętość musi być przekazana do jednostek nadrzędnych lub zarządzania przedsiębiorstwem wydobywczym, a także od wymagań dotyczących dokładności określania objętości oraz od środków finansowych przeznaczonych na ten cel. Drugorzędne warunki określania objętości obejmują: poziom szkolenia personelu w poszcze-

gólnych procedurach i metodach pomiaru i obliczania objętości, standardy techniczne przedsiębiorstwa, stosowane oprzyrządowanie, sprzęt i oprogramowanie. W artykule porównano wartości dokładnie zdefiniowanej bryły matematycznej (segment cylindryczny) z metodami obliczania objętości zwykle stosowanej w praktyce górniczej i geodezyjnej oraz programy powszechnie stosowane do obliczania objętości w celu określenia wartości progowej odchylenia systematycznego w pomiarach wejściowych do określenia objętości. Model matematyczny jest podstawą do określenia prawidłowych objętości wydobytego materiału. Proces określania błędów objętościowych w modelu matematycznym został zweryfikowany na rzeczywistym składowisku węgla. Porównanie błędów między cyfrowym modelem matematycznym a rzeczywistym składowiskiem węgla homogenizacyjnego przedstawiono na końcu artykułu.

Słowa kluczowe: określenie objętości; wydobywanie węgla; wybór metody

