GOSPODARKA SUROWCAMI MINERALNYMI

Tom 21 2005 Zeszyt 4

BOŻENA SKORUPSKA*, DANIEL SARAMAK**

Methods of determination the optimal quality of the concentrate according to chosen technological and economic criteria

Key words

Copper ore enrichment, optimisation, mathematical modelling, assessment of enrichment processes

Abstract

In the article there are presented chosen technological and economic criteria of assessment of mineral processing processes on the example of copper enrichment. Based on these criteria it is possible to build a model optimising the work of KGHM "Polska Miedź" S.A., which takes into consideration both changeable ore parameters and technology of enrichment applied in the plant. Specific solution of the model can be obtained with using computational programs based on mathematical programming methods.

Introduction

In the enrichment process the feed is divided into mainly two products: the concentrate with high content of useful mineral and the tailings in which the content of useful mineral is as low as possible. Industrial needs caused recently an intensive development in the group of technological assessment methods concerning the efficiency of enrichment process. Such methods are used to judge the quality of feed, products or the processes. They are also the

* M.Sc. Eng., Institute of Non-Ferrous Metals; Mineral Processing and Secondary Materials Utilization Department; Gliwice, Poland.

** D.Sc. Eng, Department of Mineral Processing Environmental Protection and Waste Utilisation, Faculty of Mining and Geoengineering, AGH University of Science and Technology, Kraków, Poland.

Reviewed by Prof. Marian Brożek

basis of mineral processing plant work assessment. Technological evaluation of the efficiency of industrial and laboratory separation processes is based on the analysis of average content of useful mineral in the feed and in products, that is α, β and   indexes. Main combinations of such indexes in formulas describe yield of products, the recovery and losses of ingredients. Technological enrichment indexes are useful for technologist, when one dependent variable is analysed as a function of other index playing the role of independent variable.

Technological and economic criteria are starting point for building suitable model optimising work of mineral processing plant, what is presented in the article. Solution of that model can be obtained with using method of mathematical programming.

1. Technological criteria of the process efficiency

The basic criterion of technological assessment of enrichment process is an efficiency of enrichment resulting from Hancock's index E_a (Stępiński 1964) and defined by Stępiński as *absolute enrichment coefficient*, presented in formula (1)

$$E_a = 100\frac{\gamma(\beta \quad \alpha)}{\alpha \;\; 100 \quad \dfrac{\alpha}{\beta_t}} = 100\frac{\varepsilon \quad \gamma}{100 \quad \gamma_t}\,[\%] \tag{1}$$

and the Taggart's formula (Taggart 1956):

$$T = \gamma\,\frac{\beta \quad \alpha}{\alpha} \tag{2}$$

where:

- β_t — the content of useful mineral in the theoretical "pure" concentrate (in which there are only compounds of useful mineral) or just in the real mineral. In the event of existing of several minerals (i.e. metal deliverers), β_t is a weighted average of metal content in all component minerals,
- γ_t — theoretical yield of such "pure" concentrate, $\gamma_t = 100\alpha/\beta_t$

From the technological point of view there is also significant index describing *losses of metal per growth of useful mineral content in the concentrate* (Madej 1978):

$$G_w = \frac{\quad}{\beta \quad \alpha} = 100\frac{\quad}{\alpha(\beta \qquad)} \tag{3}$$

The assessment of technological efficiency of enrichment processes can be also done on the basis of the recovery criterion for divided components in appropriate products E (formula (4)) (so-called efficiency based on the profit in separated products):

$$E = \varepsilon(100 - R) \tag{4}$$

or on the basis of selectivity coefficient E_s:

$$E_s = \frac{\beta(1\quad)}{(1\quad\beta)} = \frac{\varepsilon(1\quad R)}{R(1\quad\varepsilon)} \tag{5}$$

where:

ε — recovery of metal in the concentrate [%],

R — recovery of tailings in the concentrate [%].

Technological coefficients of technological assessment analysed above apply both to one- and multi-metallic ores. However, in enrichment processes of multi-metallic ores technological efficiency criteria are presented separately for each component. The main disadvantage of such approach is that it is assumed equality of all components, what occurs rarely in the reality. It may lead to distortion of assessment of technological efficiency.

1.1. Optimisation of enrichment process with using technological criteria

In order to characterize an optimal content of copper in concentrates produced in mineral processing plants, there were running industrial investigation over concentrates and tailings

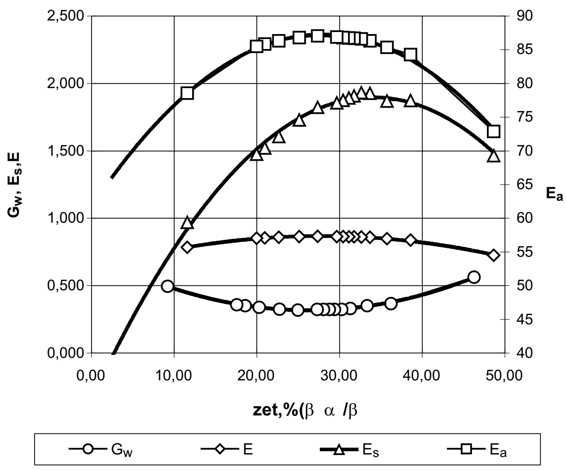

Fig. 1 Enrichment curves for ore from Rudna district (in $\beta - E_a$, $\beta - E$, $\beta - E_s$, $z - G_w$ co-ordinates systems), where: E_a — absolute enrichment coefficient, G_w — index of metal losses, E — "efficiency to recovery" coefficient, E_s — selectivity index

Rys. 1. Krzywe wzbogacania dla rudy z rejonu Rudna (w układzie współrzędnych $\beta - E_a$, $\beta - E$, $\beta - E_s$, $z - G_w$), gdzie: E_a — absolutny współczynnik wzbogacania, G_w — wskaźnik strat metalu, E — wskaźnik „efektywność/uzysk", E_s — wskaźnik selektywności coming from three districts of OZWR, namely: from Rudna district, from Polkowice district and from Lubin district. There were carried out flotation analyses of such products with using the fractional flotation method (similar to Dell's technique). Based on these analyses and computed enrichment indexes, there were drew suitable curves, presented in the Fig. 1. Approximation of presented curves with using a parabola function allows to characterize the most profitable content of copper in the concentrate according to chosen optimisation criterion. Results of such approximation are presented in Table 1.

TABLE 1

Computed values of $\beta_{optimal}$ according to chosen efficiency criteria (denotations the same like for Fig.1)

TABELA 1

Obliczone wartości $\beta_{optimal}$ według wybranych kryteriów efektywności (oznaczenia jak na rys. 1)

Type of the ore	$\beta_{optimal}$			
	E_a	G_w	E	E_s
Rudna district (production processes for sandstone)	27.87	28.9	29.5	34.63
Polkowice district	22.05	25.15	20.7	26.81
Lubin district	13.22	15.34	13.20	16.53

2. Economic criteria

In connection with the fact that technological efficiency criteria do not fill in the problem of efficiency process assessment, *economic assessments of process efficiency* are main supplements of such coefficients. They use definitions of price, cost, profit or profitability and they are using among others: to characterize the financial effect of the plant, to production planning, to characterize own costs, etc. They can exist either as compound coefficients (i.e. the cost of processing of 1 Mg of ore) or as partial ones — components of compound coefficients (i.e. comminution, grinding or classification cost).

The other group of economic criteria represent coefficients of efficiency investment assessments, taking advantage of outcomes and technological and/or economic parameters. They are divided into static and dynamic ones including the following coefficients: comparative calculation of costs or profits, profitability calculation, depreciation calculation etc.

Generally in mineral processing plant the economic efficiency can be described as a function of quality and quantity of processed ore and produced concentrate. It can be presented as in the formula (6):

$$E = f(\alpha, \beta, Q_p, Qk, C_k, k_j) \tag{6}$$

where:

- α, β — content of copper in the feed and in the concentrate,
- Q_p, Q_k — processing of feed mass; production of the concentrate,
- C_k — price of 1 Mg of concentrate,
- k_j — enrichment cost per unit.

Economic criteria of efficiency assessment are also connected with optimisation issues, and the problem of optimisation of the enrichment processes should be considered based on the correct formula of a target function (an optimisation criterion), which may be formulated variously, depending on the approaching to the problem (i.e. Trybalski 2002; Tumidajski and others 2004).

2.1. The profit criterion

The universal optimisation criterion is the profit one, defined as:

$$Z = S \quad K \qquad \max \tag{7}$$

where:

- Z — profit of company [PLN per Mg of metal],
- S — income from selling products [PLN per Mg of metal],
- K — production costs [PLN per Mg of metal].

Profit should be calculated separately both for enrichment (Z_P) and metallurgical Z_M) stages. The profit (Z_P) per 1 Mg of ore for plant with mining and processing stages can be denoted as in formula (8):

$$Z_P = \alpha\ \varepsilon(\beta)\ C_K(\beta) - K_P = \gamma\ \beta\ C_K(\beta) - P_P \tag{8}$$

where:

- α — content of copper in the feed;
- ε — metal recovery in concentrate, described as a function of concentrate quality;
- $C_K(\beta)$ — price of metal in the concentrate [PLN per Mg],
- K_P — cost of extracting and processing of 1 Mg of feed [PLN per Mg].

In metallurgical processes the profit (Z_M) is calculated from the formula:

$$Z_M = \alpha\ \varepsilon(\beta)\ \varepsilon_M(\beta)\ [C_M - K_M(\beta)] - \alpha\ \varepsilon(\beta)\ C_K(\beta) \tag{9}$$

where:

- C_M — price of metal [PLN per Mg],

K_M — metallurgical cost per unit [PLN per 1 Mg of metal],
ε_M — recovery in metallurgical processes.

Total profit in both enrichment and metallurgical processes (Z_Σ) amounts:

$$Z_\Sigma = Z_\Sigma(\beta) = \alpha\ \varepsilon(\beta)\ \varepsilon_M(\beta)\ [C_M - K_M(\beta)] - K_P \tag{10}$$

and it is a function of copper content in the concentrate, that is $Z_\Sigma = Z_\Sigma(\beta)$.

2.2. Criterion respecting NSR formula

In many countries NSR formula (*net smelter return* or *net smelter revenue*) (Strzelska--Smakowska 1994; Paulo, Strzelska-Smakowska 1995) is applied to compute the profit of mineral processing plant. NSR formula in a simplified way allows to estimate the profit of mineral processing plant.

This method is applied to compute the income from selling basic non-ferrous metals like Cu, Zn, Sn and Ni. The NSR informs about value of concentrate produced from 1 Mg of ore and it concerns the situation when processes of extraction of ore from the ground and its enrichment takes place in one plant. The concentrate is a trade product of that plant, which is selling to the other plant — copperworks.

According to NSR method, the profit of the mine is calculating both from the formula (11):

$$\mathrm{NSR} = \mathrm{CMV} - (\mathrm{DC} + \mathrm{TC}) \tag{11}$$

where:
CMV — value of metal content in the concentrate,
DC — total shipping cost of concentrate from mine to copperworks,
TC — metallurgical cost,

and from formula (12):

$$\mathrm{NSR} = QaP - (\mathrm{DC} + \mathrm{TC}) \tag{12}$$

where:
Q — mass of the concentrate [Mg],
P — market price of metal [PLN per Mg],
a — metallurgical recovery.

The value of NSR for each combination of profit and quality of concentrate is defined as a difference between the value of metal content in concentrate (CMV value) and the total sum of shipping cost of concentrate to the copperworks (DC) and smelter charges (TC).

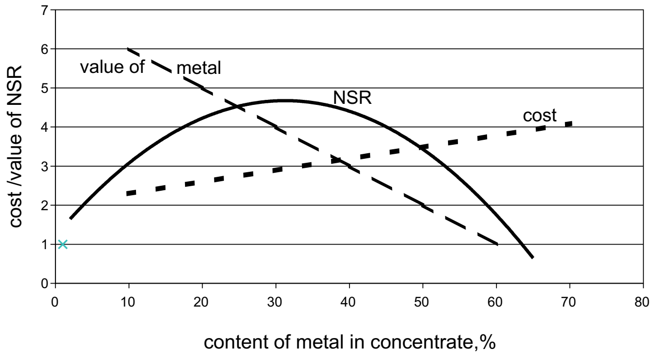

Fig. 2. Dependence of the mine profit (continuous line) on the value of metal included in the concentrate (dashed line) and the sum of shipping and the metallurgical cost (dotted line), according to NSR index

Rys. 2. Zależność zysku kopalnii (linia ciągła) od wartości metalu zawartego w koncentracie (linia przerywana) oraz od łącznych kosztów transportu do huty i przerobu metalurgicznego (linia kropkowana) według NSR

The quality of concentrate responding to the maximum value of NSR characterizes economically optimal concentrate for mineral processing plant (Fig. 2)

3. Econometric model of work of KGHM PM S.A.

Criteria of process optimisation described in section 1 are connected with the situation when given concentrate is processed in one copperworks and the income from selling is gained from only one metal. Real situation of KGHM is different as the company is a system of many plants, which cooperate in producing the final product. The quality and quantity of that product depends on efficiency of the whole system. Distribution of suitable concentrate from a plant to a copperworks is the result of its quality as well as the content of lead and arsenic and also the technology of metallurgical processing. Ore processed in KGHM is a multi-component one, the income of company comes from selling not only the base metal — copper, but also from selling silver and other accompanying metals. In order to take into consideration above situation to work out the method of determining suitable quality of concentrate, which optimises the plant's profit, appropriate models in the field of mathematical programming theory were used. Such models respect also methods of distribution of concentrates to appropriate copperworks and income resulting from selling both copper and silver.

3.1. Assumptions for building the model

In the Table 2 there are presented denotations used in building the model in description of enrichment plant's work, while Table 3 presents denotation used in description of metallurgical stage concerning distribution of concentrates to specific copperworks.

TABLE 2

Denotations used in description of enrichment stage

TABELA 2

Oznaczenia w opisie etapu wzbogacania

The district	Amount of ore processed [Mg]	Quality of concentrate β	Amount of concentrate produced [Mg]
Rudna	Q_R	β_R	$Q_R\gamma_R$
Polkowice	Q_P	β_P	$Q_P\gamma_P$
Lubin	Q_L	β_L	$Q_L\gamma_L$

TABLE 3

Denotations used in description of metallurgical stage

TABELA 3

Oznaczenia w opisie etapu przerobu metalurgicznego

Copperworks	Amount of concentrate processed in adequate copperworks [Mg]		
	from Rudna	from Polkowice	from Lubin
Legnica	$Q_{RL}\gamma_R$	—	$Q_{LL}\gamma_L$
Głogów I	$Q_{RGI}\gamma_R$	$Q_{PGI}\gamma_P$	$Q_{LGI}\gamma_L$
Głogów II	$Q_{RGII}\gamma_R$	$Q_{PGII}\gamma_P$	—

3.2. Building of model

The target function in optimization model is the profit, and the subject of optimization is to maximize this profit:

$$Z = S - K \qquad \max \tag{13}$$

The profit of company is a difference between incomes from selling of metals (*S*) and cost of production (*K*). Let's describe now both elements of profit.

3.2.1. Incomes

Income of company from selling copper and silver can be denoted as:

$$S = (M_L + M_{GI} + M_{GII})\, C_M + (S_L + S_{GI} + S_{GII})\, C_S \tag{14}$$

where:

M_L, M_{GI}, M_{GII} — amount of copper produced by copperworks Legnica, Głogów I and Głogów II respectively. Next, these quantities can be denoted with following formulas:

$$M_L = Q_{RL}\beta_R\gamma_R\varepsilon_{HL} + Q_{LL}\beta_L\gamma_L\varepsilon_{HL} \tag{15}$$

$$M_{GI} = Q_{RGI}\beta_R\gamma_R\varepsilon_{HGI} + Q_{PGI}\beta_P\gamma_P\varepsilon_{HGI} + Q_{LGI}\beta_L\gamma_L\varepsilon_{HGI}$$

$$M_{GII} = Q_{RGII}\beta_R\gamma_R\varepsilon_{HGII} + Q_{PGII}\beta_P\gamma_P\varepsilon_{HGII}$$

where:

S_L, S_{GI}, S_{GII} — amount of silver produced by copperworks Legnica, Głogów I and Głogów II respectively. These quantities can be also denoted with next formulas:

$$S_L = Q_{RL}(a_R\beta_R + b_R)\gamma_R\varepsilon'_{HL} + Q_{LL}(a_L\beta_L + b_L)\gamma_L\varepsilon'_{HL} \tag{16}$$

$$S_{GI} = Q_{RGI}(a_R\beta_R + b_R)\gamma_R\varepsilon_{HGI} + Q_{PGI}(a_P\beta_P + b_P)\gamma_P\varepsilon_{HGI} + Q_{LGI}(a_L\beta_L + b_L)\gamma_L\varepsilon_{HGI}$$

$$S_{GII} = Q_{RGII}(a_R\beta_R + b_R)\gamma_R\varepsilon_{HGII} + Q_{PGII}(a_P\beta_P + b_P)\gamma_P\varepsilon_{HGII}$$

Finally, income can be denoted as in formula:

$$\begin{gathered}
S = (Q_{RL}\beta_R\gamma_R\varepsilon_{HL} \quad Q_{LL}\beta_L\gamma_L\varepsilon_{HL} \quad Q_{RGI}\beta_R\gamma_R\varepsilon_{HGI} \quad Q_{PGI}\beta_P\gamma_P\varepsilon_{HGI} \\
Q_{LGI}\beta_L\gamma_L\varepsilon_{HGI} \quad Q_{RGII}\beta_R\gamma_R\varepsilon_{HGII} \quad Q_{RGII}\beta_R\gamma_R\varepsilon_{HGII})C_M \\
[Q_{RL}(a_R\beta_R \quad b_R)\gamma_R\varepsilon_{HL} \quad Q_{LL}(a_L\beta_L \quad b_L)\gamma_L\varepsilon_{HL} \quad Q_{RGI}(a_R\beta_R \quad b_R)\gamma_R\varepsilon_{HGI} \\
Q_{PGI}(a_P\beta_P \quad b_P)\gamma_P\varepsilon_{HGI} \quad Q_{LGI}(a_L\beta_L \quad b_L)\gamma_L\varepsilon_{HGI} \\
Q_{RGII}(a_R\beta_R \quad b_R)\gamma_R\varepsilon_{HGII} \quad Q_{PGII}(a_P\beta_P \quad b_P)\gamma_P\varepsilon_{HGII}]C_S
\end{gathered} \tag{17}$$

In the formula (17) there exist following parameters:

ε_{HL} — metallurgical recovery for Legnica copperworks,
ε_{GI} — metallurgical recovery for Głogów I copperworks,
ε_{GII} — metallurgical recovery for Głogów II copperworks,
ε'_{HL} — metallurgical recovery of silver for Legnica copperworks,

ε'_{GI} — metallurgical recovery of silver for Głogów I copperworks,
ε'_{GII} — metallurgical recovery of silver for Głogów II copperworks,
C_M — price of copper on London Metal Exchange [PLN],
C_S — price of silver on exchange in New York [PLN],
$a_R, a_P, a_L; b_R, b_P, b_L$ — parameters in equations describing dependencies between contents of copper and silver in concentrates (see point 3 section 3.2.3)

There are also independent variables in the formula (17):
$\gamma_R Q_{RL}$ — mass of concentrate delivered from Rudna to Legnica copperworks [Mg],
$\gamma_L Q_{LL}$ — mass of concentrate delivered from Lubin to Legnica copperworks [Mg],
$\gamma_R Q_{RGI}$ — mass of concentrate delivered from Rudna to Głogów I copperworks [Mg],
$\gamma_P Q_{PGI}$ — mass of concentrate delivered from Polkowice to Głogów I copperworks [Mg],
$\gamma_L Q_{LGI}$ — mass of concentrate delivered from Lubin to Głogów I copperworks [Mg],
$\gamma_R Q_{RGII}$ — mass of concentrate delivered from Rudna to Głogów II copperworks [Mg],
$\gamma_P Q_{PGII}$ — mass of concentrate delivered from Polkowice to Głogów II copperworks [Mg],
β_R — content of copper in concentrate produced in Rudna district [%],
β_P — content of copper in concentrate produced in Polkowice district [%],
β_L — content of copper in concentrate produced in Lubin district [%],
γ_R — yield of concentrate produced in Rudna district [%],
γ_P — yield of concentrate produced in Polkowice district [%],
γ_L — yield of concentrate produced in Lubin district [%].

3.2.2. Cost

Total copper production cost is described in formula (18)

$$K = K_G + K_P + K_H \tag{18}$$

where:
K_G — cost of mining stage [PLN],
K_P — cost of enrichment stage [PLN],
K_H — total cost of metallurgical stage [PLN].

$$K_G = (Q_R + Q_L + Q_P)k_{jG}$$

$$K_P = Q_R\gamma_R k_{jPR} + Q_L\gamma_L k_{jPL} + Q_P\gamma_P k_{jPP}$$

$$\begin{aligned} K_H = {} & (Q_{RL}\gamma_R\beta_R k_{jHL} + Q_{LL}\gamma_L\beta_L k_{jHL}) + \\ & + (Q_{RGI}\gamma_R\beta_R k_{jHGI} + Q_{PGI}\gamma_L\beta_P k_{jHGI} + Q_{LGI}\gamma_L\beta_L k_{jHGI}) + \\ & + (Q_{RGII}\gamma_R\beta_R k_{jHGII} + Q_{PGII}\gamma_P\beta_P k_{jHGII}) \end{aligned}$$

where:

Q_R, Q_P, Q_L — denotations like in Table 2,
kj_G— unit cost of mining stage [PLN per Mg of ore],
kj_{PR} — unit processing cost in Rudna district,
kj_{PP} — unit processing cost in Polkowice district,
kj_{PL} — unit processing cost in Lubin district,
kj_{HL} — unit metallurgical processing cost in Legnica copperworks [PLN per Mg],
kj_{HGI} — unit metallurgical processing cost in Głogów I copperworks [PLN per Mg],
kj_{HGII}— unit metallurgical processing cost in Głogów II copperworks [PLN per Mg],
The other denotations like in formula (17).

Finally, cost can be dented as in the formula (19)

$$
\begin{gathered}
K=(Q_R \quad Q_P \quad Q_L)k_{jG} \quad Q_R\gamma_R k_{jPR} \quad Q_P\gamma_P k_{jPP} \quad Q_L\gamma_L k_{jPL} \\
Q_{RL}\gamma_R \beta_R k_{jHL} \quad Q_{LL}\gamma_L \beta_L k_{jHL} \quad Q_{RGI}\gamma_R \beta_R k_{jHGI} \quad Q_{PGI}\gamma_P \beta_P k_{jHGI} \\
Q_{LGI}\gamma_L \beta_L k_{jHGI} \quad Q_{RGII}\gamma_R \beta_R k_{jHGII} \quad Q_{PGII}\gamma_P \beta_P k_{jHGII}
\end{gathered}
\tag{19}
$$

Formula (13) can be then denoted as a sum of formulas (17) and (19):

$$
\begin{gathered}
Z=(Q_{RL}\beta_R\gamma_R\varepsilon_{HL} \quad Q_{LL}\beta_L\gamma_L\varepsilon_{HL} \quad Q_{RGI}\beta_R\gamma_R\varepsilon_{HGI} \quad Q_{PGI}\beta_P\gamma_P\varepsilon_{HGI} \\
Q_{LGI}\beta_L\gamma_L\varepsilon_{HGI} \quad Q_{RGII}\beta_R\gamma_R\varepsilon_{HGII} \quad Q_{RGII}\beta_R\gamma_R\varepsilon_{HGII})C_M \\
[Q_{RL}(a_R\beta_R \quad b_R)\gamma_R\varepsilon_{HL} \quad Q_{LL}(a_L\beta_L \quad b_L)\gamma_L\varepsilon_{HL} \quad Q_{RGI}(a_R\beta_R \quad b_R)\gamma_R\varepsilon_{HGI} \\
Q_{PGI}(a_P\beta_P \quad b_P)\gamma_P\varepsilon_{HGI} \quad Q_{LGI}(a_L\beta_L \quad b_L)\gamma_L\varepsilon_{HGI} \\
Q_{RGII}(a_R\beta_R \quad b_R)\gamma_R\varepsilon_{HGII} \quad Q_{PGII}(a_P\beta_P \quad b_P)\gamma_P\varepsilon_{HGII}]C_S \\
[(Q_R \quad Q_P \quad Q_L)k_{jG} \quad Q_R\gamma_R k_{jPR} \quad Q_P\gamma_P k_{jPP} \quad Q_L\gamma_L k_{jPL} \\
Q_{RL}\gamma_R \beta_R k_{jPL} \quad Q_{LL}\gamma_L \beta_L k_{jHL} \quad Q_{RGI}\gamma_R \beta_R k_{jHGI} \quad Q_{PGI}\gamma_P \beta_P k_{jHGI} \\
Q_{LGI}\gamma_L \beta_L k_{jHGI} \quad Q_{RGII}\gamma_R \beta_R k_{jHGII} \quad Q_{PGII}\gamma_P \beta_P k_{jHGII}]
\end{gathered}
$$

3.2.3. Limitations

1. Technology of metallurgical processing.

It is established a quality limitation of concentrate from the point of view of metallurgical process's technology. The content of copper in concentrate must be higher than minimum, determined by the technology of metallurgical processing:

$$\beta > \beta_{min}$$

where β_{min} — is the value determined by metallurgical process's restrictions.

2. Relationship between quality of concentrate and its yield.

the yield formula $\gamma(\beta)$ can be well characterized with using following type of function:

$$\gamma = \frac{a_1}{\beta} \quad b_1 \tag{20}$$

where: a_1, b_1 — parameters.

There are two crucial points of that hyperbola function: first one, where yield equals 100 (for $\beta = \alpha$) and the second one where value of yield is determined for β_{teor} called "best theoretical quality of concentrate". It equals respectively: for Lubin district $\beta_{teor} = 52.14\%$, for Polkowice district $\beta_{teor} = 67.73\%$, and for Rudna district $\beta_{teor} = 63.45\%$ (Saramak 2003; Saramak, Tumidajski 2004). After determine co-ordinates of these two points it is possible to compute parameters *a* and *b*. It can also be done with using the least squares method for gained technological data sets (i.e. enrichment effects).

Laboratory experiments of ore enrichment show that well enough approximation of description of dependencies between quality of concentrate and its yield can be also denoted with using exponential function:

$$\beta = a_2 \gamma^{b_2} \tag{21}$$

where: a_2, b_2 — parameters.

3. Relationships between content of copper and silver in concentrates.

In experiments lead in Institute of Non-Ferrous Metals (e.g. IMN 5035/94) over enrichment-ability of silver it was proved that for each enrichment plant district (i.e. Lubin, Polkowice, Rudna) the relationship between the content of copper and silver in concentrate can be presented with following formula:

$$\beta_{AgR} = a_R\beta_R + b_R \tag{22}$$

$$\beta_{AgP} = a_P\beta_P + b_P$$

$$\beta_{AgL} = a_L\beta_L + b_L$$

where:

$\beta_{AgR,P,L}$ — contents of silver in concentrates from Rudna, Polkowice and Lubin respectively,

β — content of copper in concentrate,

$a_{R,P,L}$; $b_{R,P,L}$ — parameters.

4. Copperworks economic capacities

Total amount of produced concentrates cannot exceed economic capacity of copper-works. We can then denote:

$$\gamma_R Q_R + \gamma_R Q_P + \gamma_L Q_L < M_1 + M_2 + M_3$$

$$Q_R = \gamma_R(Q_{RL} + Q_{RGI} + Q_{RGII})$$

$$Q_P = \gamma_P(Q_{PGI} + Q_{PGII})$$

$$Q_L = \gamma_L(Q_{LL} + Q_{LGI})$$

where:

γ — yield of concentrate [%],

Q — total amount of ore processed in enrichment plants [Mg],

M — total economic capacities of copper-works [Mg].

5. Distribution of concentrates from enrichment plants to copperworks

— Distribution of concentrates to Legnica copperworks:

$$\frac{\gamma_R Q_{RL}}{\gamma_R Q_{RL} \quad \gamma_L Q_{LL}} \quad \frac{\gamma_L Q_{LL}}{\gamma_R Q_{RL} \quad \gamma_L Q_{LL}} = 1$$

— Distribution of concentrates to Głogów I copperworks:

$$\frac{\gamma_R Q_{RGI}}{\gamma_R Q_{RGI} \quad \gamma_L Q_{LGI} \quad \gamma_P Q_{PGI}} \quad \frac{\gamma_P Q_{LGI}}{\gamma_R Q_{RGI} \quad \gamma_L Q_{LGI} \quad \gamma_P Q_{PGI}} \quad \frac{\gamma_P Q_{PGI}}{\gamma_R Q_{RGI} \quad \gamma_L Q_{LGI} \quad \gamma_P Q_{PGI}} = 1$$

— Distribution of concentrates to Głogów II copperworks:

$$\frac{\gamma_R Q_{RGII}}{\gamma_R Q_{RGII} \quad \gamma_P Q_{PGII}} \quad \frac{\gamma_P Q_{PGII}}{\gamma_R Q_{RGII} \quad \gamma_P Q_{PGII}} = 1$$

These three equations take into consideration the presence of distribution ratios determining, the way of distribution of concentrates among copperworks. Appropriate indexes are presented in Table 3. Additionally, equations below present established range of distribution proportions for each index included in Table 3. Range at each index can be modified, but the new one should be compatible with real values of distribution proportions. Descriptions in formulas are the same like in formula (17):

$$0{,}2 \le \frac{\gamma_R Q_{RL}}{\gamma_R Q_{RL} \quad \gamma_L Q_{LL}} \le 0{,}4$$

$$0{,}6 \le \frac{\gamma_L Q_{LL}}{\gamma_R Q_{RL} \quad \gamma_L Q_{LL}} \le 0{,}8$$

$$0{,}2 \le \frac{\gamma_R Q_{RGI}}{\gamma_R Q_{RGI} \quad \gamma_L Q_{LGI} \quad \gamma_P Q_{PGI}} \le 0{,}4$$

$$0{,}25 \le \frac{Q_{LGI}}{Q_{RGI} \quad Q_{LGI} \quad Q_{PGI}} \le 0{,}45$$

$$0{,}25 \le \frac{Q_{PGI}}{Q_{RGI} \quad Q_{LGI} \quad Q_{PGI}} \le 0{,}45$$

$$0{,}45 \le \frac{Q_{RGII}}{Q_{RGII} \quad Q_{PGII}} \le 0{,}65$$

$$0{,}35 \le \frac{Q_{PGII}}{Q_{RGII} \quad Q_{PGII}} \le 0{,}55$$

6. Economics of copper production process in the system: mine – processing plant – metallurgical processing.

In that approach it is taken into consideration both income from selling specific amount of copper produced from concentrate with quality β in changeable non-ferrous metals market conditions, and cost of production of the copper (Fig. 3). The limitation can be denoted as follow:

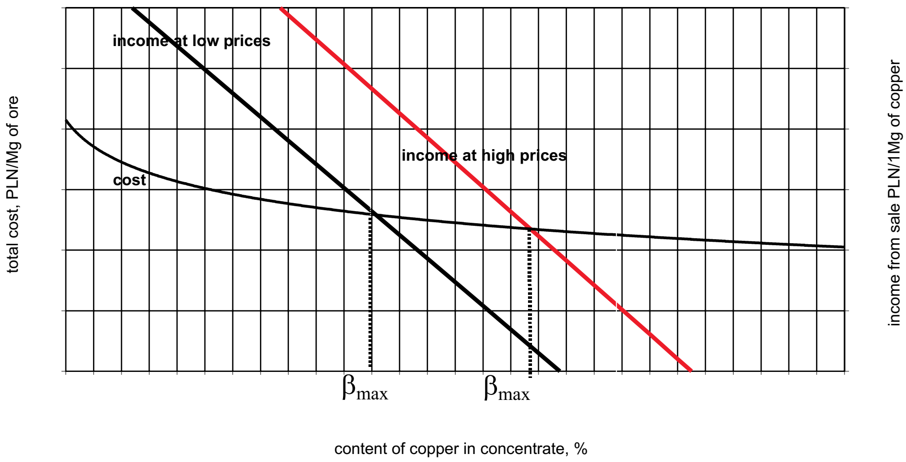

Fig. 3. The influence of limitations in quality of produced concentrates in dependence on exchange prices of copper

Rys. 3. Wpływ ograniczeń jakościowych na produkowane koncentraty w zależności od cen giełdowych miedzi

$$S > K$$

where:

S — income of company from selling of metals [i.e. in PLN per 1 Mg of copper],
K — total cost of 1 Mg of copper production [PLN per 1 Mg of copper].

Conclusions

Next stage includes searching of solutions minimizing the object function with respecting existing limitations. To reach that aim a specific computational computer software benefits form mathematical programming theory should be used. Despite the fact, that the mathematical programming theory was described in details in literature (Grabowski 1982), the complicity of the issue and its scale causes, that there exists relatively small number of computational programs enable to deal with the general problem. After specifying the problem it is possible to obtain a solution with using properly chosen computer programs based on mathematical programming theory.

The article was supported by Ministry of Science and Education, Grant no 4 T 12 A 047 28.

References

G r a b o w s k i W., 1982 — Programowanie matematyczne. PWE, Warszawa.
M a d e j W., 1978 — Ocena procesów wzbogacania. Prace Instytutu Metali Nieżelaznych t. VII, nr 3.
P a u l o A., S t r z e l s k a - S m a k o w s k a B., 1995 — Optymalizacja koncentratów rudnych metodą dochodową. I Międzynarodowa Konferencja Przeróbki Kopalin, Zakopane, Oficyna Wydawnicza Jaxa, Kraków.
S a r a m a k D., T u m i d a j s k i T., 2004 — Wpływ sterowania jakością koncentratu na efektywność procesu wzbogacania rud miedzi na podstawie OZWR KGHM „Polska Miedź" S.A. Gos. Sur. Min. t. 20, z. 3, s. 55—65.
S a r a m a k D., 2003 — Model ekonometryczny działalności KGHM „Polska Miedź" S.A. jako sposób optymalizacji procesów wzbogacania rud miedzi, Inżynieria Mineralna z. spec. 3.
S t ę p i ń s k i W., 1964 — Wzbogacanie grawitacyjne, PWN, Łódź–Warszawa–Kraków.
S t r z e l s k a - S m a k o w s k a B., 1994 — Jakość i wycena koncentratów rudnych — wpływ na gospodarkę złożem, Gos. Sur. Min.
T a g g a r t A.F., 1956 — Handbook of Mineral Dressing, J.Wiley and Sons, Inc., New York.
T r y b a l s k i K., 2002 — Analiza wskaźników jakości procesów przeróbczych jako kryteriów sterowania. Gos. Sur. Min. t. 18, z. 1, s. 25—39.
T u m i d a j s k i T., M ą c z k a W., S a r a m a k D., F o s z c z D., 2004 — Problemy optymalizacji odzysku metali w układzie kopalnia – zakład wzbogacania – huta, na przykładzie KGHM Polska Miedź S.A., ZN AGH Górnictwo i Geoinżynieria z. 2/1, s. 147—158
W i e n i e w s k i A. i zespół, 1994 — Sprawozdanie IMN 5035/94.

BOŻENA SKORUPSKA, DANIEL SARAMAK

SPOSOBY OKREŚLENIA OPTYMALNEJ JAKOŚCI KONCENTRATU WEDŁUG WYBRANYCH KRYTERIÓW TECHNOLOGICZNYCH I EKONOMICZNYCH

S ł o w a k l u c z o w e

Wzbogacanie rud miedzi, optymalizacja, modelowanie matematyczne, ocena procesów wzbogacania

S t r e s z c z e n i e

W artykule przedstawione zostały wybrane technologiczne i ekonomiczne kryteria oceny procesów wzbogacania surowców na przykładzie rud miedzi. Na podstawie przedstawionych kryteriów oceny można zbudować model optymalizujący pracę KGHM „Polska Miedź" S.A. uwzględniający zarówno zmienne parametry jakościowe rudy, jak również technologię wzbogacania stosowaną w zakładzie. Konkretne rozwiązanie uzyskać można z wykorzystaniem odpowiednich programów komputerowych z zakresu programowania matematycznego.