Application of linear geostatistics to evaluation of Polish mineral deposits

Key words
Geostatistics, evaluation, mineral deposits

Abstract
The results of geostatistical studies of the selected Polish mineral deposits have been presented. Attention has been paid to the advantages of linear geostatistics usage in the elaboration of geological data. It has been emphasized that the variability structure of mineral deposits should be analyzed more deeply and anisotropy and heterogeneity should be taken into account during the resources/reserves estimation. Geostatistics should be widely used at different stages of deposits studies.

Introduction
The major role of a geologist during a mineral deposits exploration is the estimation of geological parameters (e.g. grades, thicknesses, accumulations, and quality parameters), the mineral resources/reserves calculation, the evaluation of the accuracy of their estimation. Linear geostatistics appears to be especially attractive to solve the tasks. Numerous examples of successful applications of geostatistical methods in mineral deposits evaluation one can find in bibliography.

The basic estimation method in linear geostatistics is the ordinary kriging which enables the estimation of values of parameters with minimum error. There are two techniques of the

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Reviewed by Prof. Marek Nieć
estimation depending on a dimension of a domain to be evaluated: point kriging and block
kriging. An explanation of the method can be found in numerous geostatistical books (e.g.

Geostatistical methods allow to obtain more precise information about variabilities of
studied variables, and thus more accurate estimates of point or mean values of the variables
over limited domains. In kriging, the data about the variability structure of considered
variables are taken into account through variograms which are used to quantify spatial
correlations between observations. Kriging provides a measurement of the accuracy of the
estimates in the form of a kriging variance. This is one of the advantages of geostatistics over
traditional methods of resources/reserves assessment.

The following paper presents the results of geostatistical studies of the selected mineral
deposits in Poland.

Since 1986 the author of the paper has been involved in the studies of a practical usage of
linear geostatistics to the evaluation of the mineral deposits. The studies aimed mainly at the
kriging application to the standard resources/reserves estimation.

1. Structural analysis

The issue of the variabilities of geological parameters, especially the parameters used in
the assessment of mineral resources/reserves, plays a crucial role in a geological and mining
activity. The mathematical description of variabilities is inevitable for solving numerous
tasks. The recognition of variabilities structure of the variables enables the proper selection
of the resources/reserves calculation method and facilitates the further application of kriging
into the estimation of quality parameters. The necessary condition of the successful ap-
lication of kriging is the non-random character of the variabilities of the variables, which
finds expression in the autocorrelation between observations. In the case of an absence of the
autocorrelation, the results of the application of kriging do not deviate from those obtained
from the arithmetic mean method. It is sufficient then to apply one of the methods based on
the arithmetic mean to calculate the resources/reserves. It is also useless to do the plotting of
contour maps of geological and economic parameters (e.g. cut of grades) when their
variabilities are random.

Geostatistical studies have been undertaken for different types of mineral deposits. Par-
ticularly lead — zinc ore, copper — silver ore, native sulphur, hard coal, lignite coal, natural
aggregates, limestones deposits have been analysed.

The carried out studies prove the existence of a substantial spatial correlation in the
variabilities of thicknesses, grades, accumulations as well as in many cases of the variabilities
of many quality parameters. The studies point to the possibility of an effective application of
both geostatistics and kriging to the evaluation of mineral deposits. The observed ranges of
an autocorrelation on variograms prove the possibility of the kriging application at different
stages of deposits exploration.
Attention should be paid to the fact of a great variety of variograms models of geological parameters of the studied deposits. It reflects a complex variabilities structure of the deposits within which even particular parts can have their own features. The diversity of variograms frequently finds its explanation in geology and remains in relation with the deposits forming processes (e.g. Kokesz 1991; Nieć, Mucha, Kokesz 1988; Namysłowska-Wilczyńska 1988, 1993). The varied nature of such processes and resulting variograms makes it impossible to draw final conclusions about the relationship between them.

The studies indicate the necessity of a more thorough analysis of the variabilities structure of geological parameters, parameters used in assessment of resources/reserves, and also in-depth interpretation of the results of the analysis.

It is interesting to note that the variability of the parameters can be marked by anisotropy and heterogeneity. Therefore, the resources/reserves assessment ought to be preceded by the recognition of anisotropies and homogeneity. When anisotropy and homogeneity are evident, it should be taken into account in a kriging plan.

The recognition of anisotropy requires the sufficient number of data, and thus it is most frequently possible for the study of either large or detailed explored deposits. When a deposit is small and poorly documented by too few numbers of drills, especially spaced on a non-systematic grid, the recognition of directional variability is practically impossible. Figure 1 presents the examples of directional variograms constructed in horizontal spaces. It is worth pointing out that the directional differentiation of the variabilities of geological parameters in the horizontal spaces in the deposits is marked clearly at much longer distances, whereas at very small distances it can be assumed to be isotropic. Such observations (e.g. Kokesz 1991, 1993) lead to the conclusion that during the calculation of the resources/reserves in two-dimensional horizontal spaces, the quasi-isotropic conditions in small blocks can be most often assumed. Thus, mean variograms (omni-directional variograms) can be used in calculations. Frequently, a significant anisotropy can only be seen in the extended parts of deposits, often in blocks of higher than 200 × 200 m. The observed anisotropy has most frequently its explanation in geological processes that form a deposit.

In the studies of the already mined deposits or large detailed explored deposits, when there is a large number of data, the analysis of the stability of variograms used for resources/reserves calculations within all deposits is justified. The existence of different variograms in various zones of a deposit points to the non-homogeneity of a deposit.

The results of the geostatistical studies imply that the assumption of the concept of stability of the variabilities structure of parameters within the whole deposit is frequently not satisfying (Fig. 2). It has already been emphasised that variograms often can not be considered to be constant for a whole deposit (e.g. Kokesz 1990, 1991; Nieć, Mucha, Kokesz 1988; Kokesz, Mucha 1986; Mucha 2002). In some cases, different variograms of the same geological characteristic in various parts of a deposit can be observed. The differentiation can concern particular panels, benches, drawing levels, determined seams in stratiform deposits, and often does not find justification in a proportional effect. The proportional effect exists
Fig. 1. Examples of directional horizontal variograms
A — seam thickness no. 510 in Kazimierz-Juliusz hard coal mine; B — sulphur accumulation for Jeziórko native sulphur deposit (IX mine field); C — thickness of the Karpniki feldspar deposit

Rys. 1. Przykłady wariogramów kierunkowych obliczonych w płaszczyźnie poziomej
A — miąższości pokładu węgla 510 w KWK Kazimierz-Juliusz; B — zasobność złoża siarki rodzimej Jeziórko (pole eksploatacyjne IX); C — miąższość złoża kopaliny skaleniowej Karpniki
when there is a relationship between dispersion variance and mean value of the studied parameter (Journel, Huijbregts 1978).

If the proportional effect exists, it is justified to estimate the resources/reserves by kriging, based on relative variograms constructed for the whole deposit. Otherwise, it is advisable to carry out calculations independent in each homogeneous zone taking into account variograms developed for the zone. This should result in the improvement of the effectiveness of the evaluation of deposits.

Non-homogeneity may also appear in the presence of a drift and differentiation values of geological parameters in averages. However, most frequently, the drift can be observed on extended areas. In general, small parts of deposits may be considered to be quasi-homogeneous (e.g. Kokesz 1991, 1993; Namysłowska-Wilczyńska 1988, 1993). Thus, the ordinary kriging
can be used to provide local kriged estimates within each zone or neighbourhood of quasi-homogeneity.

The studies imply that the analysis of the results of a deposit exploration should embrace the estimation of its homogeneity. The study of non-homogeneity requires sufficient data, thus it is possible only at the stage of the study of large, detailed explored, and especially mined deposits. The detection of non-homogeneity is often possible thanks to histograms. Therefore, the study of statistical recognition of results should precede geostatistical modelling spatial variabilities. The division of a deposit into homogeneous parts ought to be based on geological criteria. As results from the carried out studies, the observed non-homogeneity of the data collected during the exploration and mining deposits, most frequently finds its explanation in geology (Kokesz 1990, 1991; Nieć, Mucha, Kokesz 1988; Mucha, Szuwarzyński 1994). In terms of the analysis of non-homogeneity, isoline maps of various parameters and trend maps may be also helpful.

2. Estimation of resources/reserves

Point kriging enables the prediction of values of various characteristics at nodes of a regular grid within a deposit. Therefore it can be applied as an interpolation procedure for contour mapping. Kriging is shown to be an exact interpolator which gives estimates with minimum error and thus to allows to predict values more accurate than any other interpolation procedures.

So far, the point kriging has been used for the delimitation of the deposits and areas in which specific types of resources/reserves (economic, subeconomic) or different types and qualities of the mineral raw materials occurred (e.g. Kokesz, Dolik, Rolewicz 1989; Mucha, Kokesz, Dolik 1994). Variograms studied characteristics were analysed before contour mapping and delimitation of a deposit. In the case of random variabilities of parameters, construction of isoline maps and delimitation of the deposit by interpolation were abandoned. It is worth noting that in practice the character of a spatial variability is often neglected. It leads to obvious errors e.g., a construction of isoline maps for parameters which are, in fact, random variables. Such maps are essentially useless but give a very suggestive image of the distribution of geological parameters which may mislead a mining engineer.

Geostatistics has also enabled the determination of extrapolated boundaries, accuracy of parameters and resources/reserves estimation in such defined outlines. This provided more precise information of the real deposit limits. Uncertainty about deposit boundaries was demonstrated by the maps of errors of estimates or maps showing confidence interval of exact limits of the mineralization (minimum and maximum probable extent of mineralization).

Computer-based processing of geological data allows the application of geostatistics not only to the construction of isoline maps, to the interpretation of the boundaries of a deposit, but also to the resources/reserves calculation. In the evaluation of Polish mineral deposits the
point kriging and block kriging have been applied. However, the block kriging has been used most frequently, as it enabled the direct estimates of mean values of geological parameters and also the accuracy of the estimates for any area of a deposit. The kriging has been also adopted to traditional methods of the resources/reserves calculation.

For the resources/reserves calculations, the following classical methods have been used: geological and exploitation blocks, polygons, geological sections, isolines, an ordinary arithmetic mean. The ordinary arithmetic method is commonly used in a global resources/reserves calculation when a whole deposit is regarded as one block. The choice of a method and manner of a deposit subdivision into calculation blocks largely depends on the geological structure of a deposit and the type of an exploratory system.

There have been different ways of the application of kriging to resources/reserves calculations. Most frequently, it was used for the estimation of mean values of geological parameters (grades, thicknesses, accumulations) and tonnages in particularly selected blocks. The total resources/reserves for a whole deposit were determined as a sum of those from all blocks. The 2D kriging was mainly applied in order to supply estimates relative to a deposit in horizontal or vertical two-dimensional spaces.

For stratiformed sub-horizontal deposits explored by vertical drills, 2D kriging was generally used for the resources/reserves calculations in single panels determined on the basis of geological blocks. This method of an estimation was applied to the deposits of copper-silver ore (Kokesz 1998), hard coal, (e.g. Kokesz 1993; Mucha, Kokesz 1986), lignite coal (e.g. Kozula, Mazurek 1996), sulphur (e.g. Kokesz 1991; Kokesz, Krysiak, Dolik 1994), natural aggregate (e.g. Kokesz, Dolik, Rolewicz 1989). The estimation of the effectiveness of this method of calculations was also the subject of separate studies. Results of the studies have proved the effectiveness and usefulness of kriging. For example, it has been pointed out that in unfolded hard coal seams, kriging ensures accuracy of the resources/reserves estimations in single blocks 20—40% higher in comparison with those obtained from classic arithmetic mean method (Kokesz 1993).

In the case of a mined deposit, the kriging application has most frequently amounted to mean values of various geological characterisics and reserves estimation in single exploitation blocks (blocks delimited by mining workings). Such kriging plan was developed for the deposits of copper-silver ore (e.g. Mucha, Nieć, Szwed 2005; Namysłowska-Wilczyńska 1988, 1990, 1993), lead — zinc ore (e.g. Mucha 2002), hard coal (Kokesz 1993; Mucha, Kokesz 1986). Traditionally, in the method of exploitation blocks, mean values of geological parameters in each unit are calculated as arithmetic means. In kriging, if variability of a parameter is non-random, weights given to sample points depend on geometry of the block to be estimated and data configuration. Thus, when variability of a parameter is non-random, differentiations in reserves calculated by traditional and geostatistical methods are observed in individual mining blocks.

Native sulphur deposits are mined by underground melting (Frasch method). The detailed exploration of the deposits is made with drillholes on a grid 45—90 m. These holes are subsequently used as exploitation ones. The polygons method is generally applied for
reserves estimation. The explored deposit is divided into polygones. Each of them represents an ‘area of influence’ of an individual drillhole. The reserves are calculated separately for the polygones and then added. This method enables the current control of sulphur recovery from the individual polygons. A great disadvantage of the polygons method is an extension of a single observation on a sizable ‘area of influence’. It leads to under — or over — estimation of the reserves within the polygones and, consequently, to the errors in the estimation of the degree of local sulphur recovery. It was proposed to estimate mean values of geological parameters and reserves in an individual polygon by kriging using the data from its central drillhole and the data from surrounding, adjacent drillholes (Kokesz 1991). It was found that kriging allows to estimate the sulphur tonnage for each polygon with the accuracy about twice obtained with the traditional polygons method.

Kriging has found its usage as a supporting procedure for the geological cross-sections classic method. The estimation of resources of Myszków massive — stockwork Mo-W-Cu deposit can be an example of such an application (Mucha, Kokesz, Dolik 1994). The deposit is characterised by a complicated shape and geological structure. The deposit was sampled by 24 drillholes situated in 6 parallel lines (Fig. 3).

Drill cores were cut into constant lengths 0.5 m and analysed for Mo, W, Cu. As a whole, about 45 000 samples were taken. The deposit was divided into blocks constructed on the basis of geological sections. Variograms of metal grades were developed for individual sections and point kriging was used for plotting of contour maps and for delineation of the deposit in vertical sections. Resources were calculated separately for different types of ore. 2D block kriging was applied to the evaluation of mean metals grades in small 50 × 50 m units of each section (Fig. 3). Quantities of metals in the blocks constructed on the basis of geological sections were determined as the product: area in the section within which specific types of ore are contained, weighted means of metals grades in ore, ore bulk density, and breadth of a block ascribed to a section. Total resources were defined as the sum of the results obtained from the blocks. Kriging enabled more precise outlines of the deposit and more accurate resources estimation.

Point kriging can constitute a supportive procedure for a classic method of isolines. The modification of the method of resources/reserves estimation consists in the usage of point kriging for the construction of contour maps. In the case, with this application of kriging, it is practically impossible to directly determine an error of resources/reserves estimation.

The calculation of resources is comprehensive and complicated work, and thus even serious errors — mistakes in computation — are not rare. For this reason, control calculations are obligatory. It is also desirable to calculate resources by two methods and then judge the differences. Kriging can also be used as a method of verification. In this case kriging can be applied in global resources estimation. It is worth emphasising that in the estimation there is the possibility to directly evaluate the error of the global resources estimation, which is not completely possible in total resources estimation basing on local kriged estimates in points or within small blocks.
Fig. 3. Application of kriging in evaluation of Myszków Mo-W-Cu deposit
(after Mucha Kokesz, Dolik 1994)
A — map of drillholes localization: 1 — drillholes, 2 — boundaries of the deposit, 3 — cross-sections, 4 — limits of calculation blocks constructed on the basis of vertical sections, 5 — economic resources, 6 — subeconomic resources;
B — variogram of Mo grades; C — estimation of Mo grades in small units on a vertical section: 1 — drillholes, 2 — overburden, 3 — economic resources of Mo-W-Cu ore, 4 — economic resources of Cu-Mo-W ore, 5 — subeconomic resources, 6 — granitoid intrusive, 7 — contour lines of Moe grades (defined as Moe = Mo + 1.5W + 0.3Cu) obtained by point kriging, 8 — mean Mo grades estimated by block kriging

Rys. 3. Zastosowanie krigingu w ocenie zasobów złoża Mo-W-Cu Myszków
(wg Muchy, Kokesza, Dolik 1994)
A — mapa lokalizacji otworów rozpoznawczych: 1 — otwory wiertnicze, 2 — granice złoża, 3 — linie przekrojów, 4 — granice bloków obliczeniowych zasobów, przyprzekrojowych, 5 — zasoby bilansowe, 6 — zasoby pozabiliansowe;
B — wariogram zawartości Mo; C — oszacowanie zawartości Mo w elementarnych połatkach wybranego przekroju przez złoże: 1 — otwory wiertnicze 2 — utwory nadkładu, 3 — zasoby bilansowe — ruda Mo-W-Cu, 4 — zasoby bilansowe — ruda Cu-Mo-W, 5 — zasoby pozabiliansowe, 6 — granitoid, 7 — izolinie wartości Moe (wyznaczonej z formuły: Moe = Mo + 1.5W + 0.3Cu), 8 — zawartości Mo ustalone metodą krigingu blokowego
Kriging application directly to global resources assessment requires the adoption of a hypothesis about the reliability of variogram models describing variabilities of geological parameters at longer distances and also of the homogeneity within the whole deposit. The adoption of such speculations finds most frequently an explanation in preliminary stages of a deposit studies in respect of lower expectations of a degree of geological assurance of resources.

Surface geophysical measurements in geological exploration allow gaining better information about the correlation structure of a deposit. It has a vital meaning in the case of a deposit sampled by a small number of drills. Then, there is a possibility of using also geophysical studies to develop a more reliable model of variability structure of a deposit and resources estimate (Kokesz, Kotowski, Mucha 2002). The geophysical methods were introduced, for example, into the exploration of the Karpniki feldspar deposit. The deposit was sampled by 9 drillholes. The drillholes were spaced 90—250 m apart. In the geophysical studies, geoelectrical measurements were also used. As a whole, 72 measurements were carried out on the regular square 50m grid. The geostatistical elaboration of the results of the studies included a construction of models of studied parameters variograms, contour mapping by point kriging, resources evaluation by block kriging, and assessment uncertainty of calculated resources. Variograms were developed on the data obtained from drills and geophysical measurements. Geostatistical methods and geophysical studies allow us to obtain a better insight not only into the spatial correlation structure and continuity of the mineralization of the deposit but also geological structure, and mineral resources.

Fig. 4. Map of resources estimation of Basznia native sulphur deposit
1 — drillholes, 2 — main faults, 3 — boundaries of the deposit, 4 — blocks of resources calculation, 5—7 — relative error of resources estimation: 5 — 10 to 20%, 6 — 20 to 30%, 7 — above 40%

Rys. 4. Mapa obliczenia zasobów złoża siarki rodzimej Basznia
1 — otwory rozpoznawcze, 2 — główne uskoki, 3 — granice złoża, 4 — parcele obliczeniowe zasobów, 5—7 — dokładność oszacowania zasobów: 5 — 10 do 20%, 6 — 20 do 30%, 7 — ponad 40%
Kriging allows not only to gain more precise information about a distribution of resources/reserves within a deposit but also- in comparison to another method of reserves/resources calculation- enables the possibility of assessment of an accuracy of reserves/reserves estimation in individual parts of a deposit. It gives the opportunity to construct maps of error estimates (Fig. 4). The maps constitute a favourable form of analysis of a degree of geological assurance, and thus they can be used for an objective classification of resources/reserves.

**Summary and conclusions**

Geostatistics is more and more widely used in solving practical problems appearing in geological and mining activities. It can be, for example, applied to a deposit modelling and its delimitation, resources/reserves calculation, assessment of uncertainty of resources/reserves estimation, optimal drill holes and samples spacing. Geostatistical methods give more reliable information about deposit boundaries and resources/reserves.

In practice, the most frequently used kriging technique is the ordinary kriging. There can be various ways of its usage in resources/reserves calculation. The simplest is the application of two-dimensional kriging to horizontal or vertical spaces. This manner is much easier than the estimation carried out in three-dimensional space.

An effective application of this procedure in resources/reserves estimation is possible if certain requirements are met. It depends on variabilities structures of parameters under study, and also reliability of variograms used in calculations. The representativeness of the variograms is influenced by many factors, among which the number of samples, samples spacing, errors in the measuring of parameters and compiling the data, a degree of homogeneity of a deposit, and finally knowledge and experience gained in practical application of geostatistics play a crucial role. The most essential influence on the effectiveness of kriging have variability structures of parameters. The necessary condition of the successful application of kriging is non-random character of the variables. When variabilities of the characteristics are random, then kriged estimates do not differ from that obtained by arithmetic mean method.

It is thus purposeful to study the structure of the spatial variabilities of the parameters which are taken into consideration in the calculation of resources/reserves and also characteristics whose differentiation should be presented on maps. The knowledge about variograms should allow us to make decisions referring to the way of defining deposit boundaries, and resources/reserves calculations, and also decisions about the usefulness of contour mapping.

Geostatistical studies of the Polish mineral deposits prove the possibility of the kriging application at different stages of the deposits exploration.

The results of the carried out studies imply the need to deeply analyse the structure of the spatial variabilities of parameters and take into account anisotropy and heterogeneity in the resources/reserves estimation. It should contribute to the improvement of the effectiveness of estimation by kriging.
Modelling of variabilities structures of geological parameters should be preceded by a detailed statistical analysis of the available data involving the construction of histograms and correlation of diagrams. It provides the possibility of a preliminary assessment of representativeness of data, a degree of data homogeneity.

The work has been financed by the National Committee of Scientific Research (10.10.140.313/2005).

REFERENCES

Zastosowanie liniowej geostatystyki w badaniach złóż kopalń stałych w Polsce

Słowa kluczowe
Geostatystyka, szacowanie zasobów, złoga kopalń

Streszczenie
W artykule przedstawiono wyniki geostatystycznych badań złóż kopalń stałych w Polsce. Zwrócono uwagę na korzyści wynikające z zastosowania liniowej geostatystyki przy dokumentowaniu złóż. Wskazano na potrzebę głębszego analizowania struktury zmienności złoga i uwzględniania anizotropii oraz niejednorodności w szacowaniu zasobów. Metody geostatystyczne dzięki swym zaletom powinny być powszechnie wykorzystywane przy dokumentowaniu złóż.