

MARIAN BROŻEK*, ANNA MŁYNARCZYKOWSKA*

The relation between the dispersive model of the particle and the distribution of permanent adhesion rate constant in the coal flotation process

Introduction

Flotation is a mass process the course of which is determined by numerous factors of random character which characterize the material subjected to flotation, such as particle shape and size, particle hydrophobic properties and rate of covering with flotation reagent. It is supplemented by hydrodynamic conditions in the flotation chamber which are also distributed mostly by random in the chamber volume. All this makes the description of raw material flotation properties be of statistical character and the value describing these properties will be characterized by a certain distribution.

There are two aspects of investigation of this problem. When analyzing the distribution of flotation properties of the material which is homogeneous as far as geometrical and physical properties are concerned, it is assumed that the particles are spheres of the same diameter and the same hydrophobic properties. These are the so-called equal-floating particles. The distribution of flotation properties, characterized by the flotation rate constant of the 1st order, is connected with the distribution of induction time. This aspect of investigations of flotation kinetics is presented in numerous publications (Pogorelyj 1961; Woodburn, Loveday 1965; Inoue, Imaizumi 1968; Tille, Panou 1968; Kapur, Mehrotra 1974; Harris, Cuadros-Paz 1978; Geidel 1985; Xu Changliani 1985; Schulze 1992). The frequency function of the flotation rate constant is expressed by the gamma distribution.

* Professor, ** Ph.D., AGH-University of Science and Technology, Faculty of Mining and Geoinforming, Department of Mineral Processing and Environmental Protection, Kraków, Poland; e-mail: brozek@agh.edu.pl; mindziu@agh.edu.pl.

When analyzing the distribution of flotation properties of the non-homogeneous material from the point of view of flotation properties, it is necessary to combine these properties with the content of a useful component (in the case of ores) and the ash content (in the case of coal). Less attention has been paid so far to this aspect of investigation, significant from the point of view of upgrading of raw materials. This paper attempts to fill this gap.

Flotation is the process of enrichment, which consists in differentiating the content of useful component in separation products. The content of useful component is a volume property. Therefore, as a result of flotation, the differentiation of volume property occurs by the use of differentiation of surface properties. Respectively, these should be a relation between particle volume properties and particle surface ones. A hypothesis can be put forward that the distribution of surface (hydrophobic) properties of particle in the sample should be related with the distribution of volume properties. In the case of coal this relation can be deduced in the following way. The particle surface properties depend on the exposition rate of mineral matter on the particle surface which, subsequently, depends on the content of this matter in the particle. There is a following chain of dependences. The exposition rate of mineral matter on the particle surface increases with the growth of its content. On the other hand, however, the rate of covering the particle surface with a collector and adhesion effectiveness decrease; thus adhesion rate constant decreases, too. Therefore the distribution of adhesion rate constant in a sample will depend on the distribution of the mineral matter content.

Assuming that disseminations of mineral matter are evenly distributed in the volume of a spherical particle of radius R_p , the total content of mineral matter in the sphere of radius r ($r \leq R_p$) and thickness of infinitely small sphere dr will be equal to:

$$dA_m = A_m 4 \pi r^2 dr \quad (1)$$

where:

A_m – volume content of mineral matter in the particle [m^{-3}].

If $r = R_p$, it can be assumed that the exposition rate of mineral matter on the particle surface or, in other words, the surface content of mineral matter on the particle surface ω [m^{-2}] will be:

$$\omega = \frac{dA_m}{4\pi R_p^2} = A_m dr \quad (2)$$

The surface content is thus proportional to the volume content of mineral matter in the particle. The larger the surface content of mineral matter, the lower effectiveness of adhesion.

As will be showed experimentally, adhesion rate constant λ_o decreases with the growth of ash content (A), according to the following dependence:

$$\lambda_o(A) = a e^{-bA} \quad (3)$$

where a in the above formula represents adhesion rate constant for pure coal which is dependent on type of coal, degree of oxidation and previous treatment by other reagents, b – constant.

From formula (3), the reciprocal dependence, ash content on adhesion rate constant, is as follows:

$$A = \frac{1}{b} \ln \frac{a}{\lambda_o} \quad (4)$$

In the macroscopic approach the bubble mineralization is the results of two events: the collision of a particle with a bubble and the adhesion of a particle to the bubble surface. Both these events are of a random character and the process of flotation can be considered as a stochastic process. As a result of turbulence of medium in the flotation chamber, despite the process of adhesion to bubbles, there is a reverse process of lower intensity, i.e. the process of detachment the particles from the air bubbles (Mika, Fuerstenau 1968; Schulze 1977, 1992; Woodburn et al. 1971; Maksimov, Emelianov 1983; Geidel 1985; Honaker, Ozsever 2003).

From the point of view of mathematics, the stochastic process which considers the phenomenon of adhesion and detachment particles from the bubble surface is called the process of birth and death (Bailey 1964; Stachurski 1972).

1. Stochastic model of bath flotation kinetics

The number of floating particles which remains in the chamber in the free state (not coagulated with air bubbles) to the time t is a random variable. Let $N(t)$ represents a random variable denoting the number of particles which were not subjected to adhesion till the time t .

The following value:

$$P_{n+1}(t) = P[N(t) = n + 1] \quad (5)$$

is the probability of the fact that the random variable in the moment of time t assumes the value $n + 1$. Such a state of the process is denoted by $E_{n+1}(t)$. From the physical point of view it means the same as if $n + 1$ particles remained in the free state in the flotation chamber up to the time t (Bodziony 1966). After the time $t + \Delta t$ the random variable assumes the value n . This state of the process is denoted by $E_n(t + \Delta t)$. One particle underwent adhesion into the air bubble during the moment of time Δt . Thus it can be said that free particles disappear (die) from the flotation chamber, therefore the name of such a stochastic process is the death process.

The probability of passing from the state $E_{n+1}(t)$ to the state $E_n(t + \Delta t)$ is:

$$P[E_{n+1}(t) \rightarrow E_n(t + \Delta t)] = \lambda(n+1)\Delta t + o(\Delta t) \quad (6)$$

when $\lambda(n)$ is the function of intensity of passing from one state to the other, or the function of intensity of the adhesion process. Expression (6) means that the probability of decreasing by a unity the number of free particles in the time Δt is proportional to this time with accuracy to the infinitesimal value of higher order $o(\Delta t)$. In other words, the probability of an event consisting in the fact that in the time Δt more than one particle undergoes adhesion to the air bubble decreases faster to zero than the time Δt .

It is assumed for the function of process intensity that:

$$\lambda(n) = n\lambda_o \quad (7)$$

where λ_o is the constant characterizing the process. In other words, the intensity of the adhesion process is proportional to the number of particles in the free state.

Solving Kolmogorov-Feller's system of differential equations for this process, the expression for the recovery of particles which underwent adhesion in the form of kinetics equation of the first order is obtained:

$$R(t) = 1 - \exp(-\lambda_o t) \quad (8)$$

where λ_o denotes the flotation rate constant or the total adhesion rate constant.

In the above model the particles which underwent adhesion to the air bubbles are not able to return to the free state. It is, therefore, a pure death process.

On the other hand, as we know, as a result of turbulence of the medium in the flotation chamber, a process reverse to the process of adhesion occurs, i.e. the detachment of particles from the air bubbles. Therefore, the description of the flotation process by a pure death process is not sufficient. The model based on the stochastic process of birth and death (Bailey 1964; Stachurski 1972) is a more complete model, describing better the character of phenomena occurring in the flotation chamber. Therefore, in the volume of the flotation chamber, there are two mechanisms, namely of adhesion and detachment of attached particles from the air bubbles as a result of medium turbulence in the flotation chamber.

The number of particles lifted by air bubbles to the froth layer to the time t is a random variable $N(t)$. The probability of transition is defined as follows:

$$P_{jl}(t, t + \Delta t) = P[N(t + \Delta t) = l | N(t) = j] \quad 0 \leq t \leq t + \Delta t \quad (9)$$

It is a conditional probability of the fact that in the moment $t + \Delta t$ there are l of particles attached to the air bubbles if in the moment t their number was j . Assuming that the process is homogeneous in time, i.e.

$$P_{jl}(t + \Delta t) = P_{jl}(\Delta t) \quad (10)$$

the following postulates are imposed upon the probability of transition:

$$P_{jl}(\Delta t) = \begin{cases} \lambda(j)\Delta t + o(\Delta t) & \text{for } l = j+1 & (11a) \\ \mu(j)\Delta t + o(\Delta t) & \text{for } l = j-1 & (11b) \\ 1 - [\lambda(j) + \mu(j)]\Delta t + o(\Delta t) & \text{for } l = j & (11c) \\ o(\Delta t) & \text{for } l \neq j-1, j, j+1 & (11d) \end{cases}$$

where $\lambda(j)$ denotes the intensity of the adhesion process while $\mu(j)$ denotes the intensity of detachment of particles from air bubbles.

Equation (11a) denotes that the probability of increasing by unity the number of attached particles is proportional to the time Δt with accuracy to infinitesimal of the higher order. Analogically, (11b) denotes that increasing by unity the number of detached particles is proportional to the time Δt . In the state of equilibrium the process is characterized by the sum of effects of adhesion and detachment. Accordingly, expression (11c) denotes that during the time there was no change of the state of particles in the chamber. On the other hand, expression (11d) denotes that the probability stating that in the time Δt more than one particle was subjected to adhesion or was detached from the bubble is an infinitesimal value. It is obvious that $N(0) = 0$. The probability of transition $P_{ol} = P_l(t)$ fulfills of Kolgomorov-Feller's system of differential equations:

$$\frac{dP_l(t)}{dt} = -[\lambda(l) + \mu(l)]P_l(t) + \lambda(l-1)P_{l-1}(t) + \mu(l+1)P_{l+1}(t) \quad (12)$$

The above system of equations expresses the principle of probability conservation. The intensity functions of adhesion and detachment processes, by analogy with equation (7), are expressed by the following formulas:

$$\lambda(l) = \lambda_o (n_o - l) \quad (13a)$$

$$\mu(l) = \mu_o l \quad (13b)$$

Therefore the intensity of the adhesion process is proportional to the number of particles remaining in the free state ($n_o - l$) whereas the intensity of the detachment process is proportional to the number of particles attached to the bubbles (l).

After considering equations (13a) and (13b), equations (12) take the form:

$$\frac{dP_l(t)}{dt} = -[\lambda_o(n_o - l) + \mu_o l]P_l(t) + \lambda_o(n_o - l + 1)P_{l-1}(t) + \mu_o(l + 1)P_{l+1}(t) \quad (14)$$

Taking into account the initial conditions:

$$P_l(0) = 1 \quad \text{for } l = 0 \quad (15a)$$

$$P_l(0) = 0 \quad \text{for } l = 1, 2, \dots, n_o \quad (15b)$$

the system of equations (14) has the following solution:

$$P_l(t) = \binom{n_o}{l} \frac{l}{(\lambda_o + \mu_o)^{n_o}} [(\mu_o + \lambda_o)e^{-(\lambda_o + \mu_o)t}]^{n_o - l} [1 - e^{-(\lambda_o + \mu_o)t}]^l \quad (16)$$

The mean value of random variable $N(t)$ is equal to:

$$E[N(t)] = \sum_{l=0}^{n_o} lP_l(t) = \frac{n_o \lambda_o}{\lambda_o + \mu_o} [1 - e^{-(\lambda_o + \mu_o)t}] \quad (17)$$

Consequently, the recovery of particles elevated to the froth layer is expressed by the formula:

$$R(t) = \frac{E[N(t)]}{n_o} = \frac{\lambda_o}{\lambda_o + \mu_o} [1 - e^{-(\lambda_o + \mu_o)t}] \quad (18)$$

It can be said that constants λ_o and μ_o are, respectively, the constants of the adhesion and detachment of particles from the air bubbles.

According to formula (18):

$$\lim_{t \rightarrow \infty} R(t) = R_\infty = \frac{\lambda_o}{\lambda_o + \mu_o} \quad (19)$$

$$\left. \frac{\partial R(t)}{\partial t} \right|_{t \rightarrow 0} = \lambda_o \quad (20)$$

Fitting the empirical dependence of recovery upon time to the model dependence, it is possible to estimate the value of adhesion and detachment processes in given physico-chemical and hydrodynamic conditions in the flotation chamber.

2. Distribution function of adhesion rate constant

The distribution of adhesion rate constant in a sample depends on the distribution of content of mineral matter while the distribution of mineral matter content was determined according to the particle dispersion model (Brožek 1995b, c). In this model mineral raw materials are treated as a multi-phase system and a phase is a part of the medium which is homogeneous from the point of view of physical and chemical properties (Cottrell 1964). A continuous phase and a dispersed one are differentiated in the two-component dispersive system. In the case of coal the continuous phase is constituted by the organic coal matter while the dispersed phase by disseminations of mineral matter. It is assumed in the dispersive model that the number of disseminations s of the dispersed phase in the particle of volume V is a random variable $s(V)$ of Poisson's frequency function.

The cumulative distribution function of Poisson's distribution is expressed by the incomplete gamma function, also known as Pearson's function (Gradstein et. al 1971):

$$F(s) = \frac{1}{\Gamma(1+s)} \int_{nV}^{\infty} e^{-t} t^s dt = I(nV; 1+s) \quad (21)$$

where:

- $\Gamma(1 + s)$ – gamma function,
- $I(nV; 1 + s)$ – Pearson's function (Firkowicz 1970).

Subsequently, the following Pearson's function constitutes the cumulative distribution function of dispersed phase content:

$$F(A) = I(H A_p; 1 + H A) \quad (22)$$

where A_p represented the average volume content of the dispersed phase in the sample (ash content), whereas H , is a constant connected with the size of a particle and disseminations.

Substituting dependence (4) into formula (22) we obtain an expression for a cumulative distribution function of adhesion rate constant:

$$F(\lambda_o) = 1 - I\left(B_1; 1 + B_2 \ln \frac{a}{\lambda_o}\right) \quad (23)$$

where: $B_1 = H A_p$, $B_2 = H/b$ – parameters of distribution, connected with the distribution of disseminations and particle sizes.

3. Experimental method

3.1. Sample preparation

Coal type 33 was used which was crushed in a roll crusher dimensions below 0.5 mm. The wet sieve analysis was performed, achieving the (0.315–0.2) size fraction, followed by the float and sink analysis. The analysis was carried out in the zinc chloride solutions and its products were washed carefully with warm water. The following density fractions – (–1.35), (1.35–1.5), (1.5–1.7), (1.7–1.8), (1.8–2.0) (+2.0) – were selected in order to ensure the appropriate amount of the raw material for planned experiments according to the analysis of separation product yields in heavy liquids. The ash content was denoted within the obtain narrow density fractions, according to the Polish standard. The test samples were vacuum-stored to limit a possible oxidation of coal surface.

3.2. Determining the flotation kinetics

Flotation experiments for coal were carried out at room temperature in the Denver type laboratory machine of 1 dm³ chamber capacity, at the constant number of rotational speed of 2020 [min⁻¹] and fixed air consumption. The content of solids was the same in all experiments and was 80 g/dm³. Such conditions ensured the constant amount and size of air bubbles in chamber at limited turbulence of flotation pulp, caused by the rotator action. The low concentration of flotation pulp was determined by practical reasons because at low concentration of the pulp it is possible to achieve favorable results of coal flotation (Sablik 1998). Water solutions of n-butanol were used as a collecting and frothing reagent. These solutions do not change pH of the pulp but only decrease the surface tension on the gas-liquid boundary and the air dispersion in the suspension grows. Also the adsorption of alcohol on the surface of air bubbles occurs which ensures its stabilization and prevents coalescence (Malysa 2000; Krzan et al. 2002). On the basis of introductory investigations the authors chose appropriate values of reagent concentration which guaranteed the formation of froth of proper structure and durability. The samples were dried and weighed as well as the ash content was measured. The obtained results were used to determine the distribution of adhesion rate constant.

4. Distribution of adhesion rate constant

4.1. Distribution of density and ash content in the sample

A prior float and sink analysis of the coal sample in order to determine the density distribution was to be performed to determine the distribution of adhesion rate constant.

The distribution function of density in the tested sample was drawn according to the obtained result for the sample of particle size (0.2–0.315) mm in which the consecutive

densimetric fractions differ significantly in the ash content, i.e. in the content of mineral matter (which means that they should differ in their flotation properties).

The cumulative distribution function of density is expressed by the incomplete gamma function (Brożek 1995b, c):

$$F(\rho) = I[K_1; 1 + K_2(\rho - \rho_0)] \quad (24)$$

where:

K_1 and K_2 – distribution parameters.

Figure 1 presents a model cumulative distribution function of density (continuous line) together with experimental point. The equation of this distribution function in the range of particle density ($\rho_{\min}, 2.0$) Mg/m^3 is as follows:

$$F_1(\rho) = 0.83 [0.94 I(2.2; 1 + 30(\rho - 1.3))] \quad (25)$$

whereas density ρ is expressed in $[\text{Mg/m}^3]$.

Parameters K_1 i K_2 were determined by means of table of the incomplete gamma function (Pagurova 1963) by the method of gradual approaching situation in which for given value K_1 parameter K_2 reaches the value at which the index of curvilinear correlation is maximum. For dependence (25) the index of curvilinear correlation is larger than 0.95.

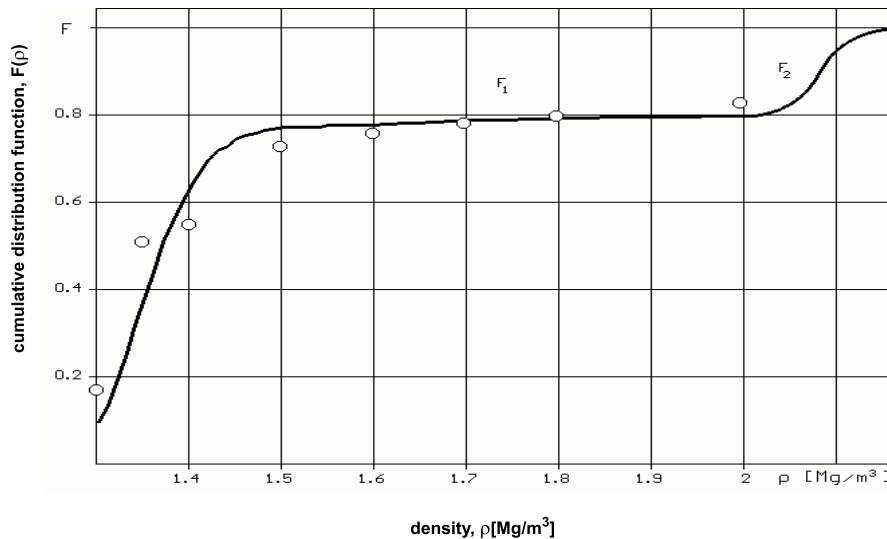


Fig. 1. Cumulative distribution function of the density in the tested sample

Rys. 1. Dystrybuanta rozkładu gęstości w badanej próbce

For the density of particles over 2.0 Mg/m^3 no detailed float and sink analysis was performed. Nevertheless, as the experiment shows (Brožek 1995b, c), the distribution of mineral matter content in this range of particle density is also expressed by the incomplete gamma function, but with the parameters different from those in the distribution below 2.0 Mg/m^3 .

Consequently, for the entire range of particle density the cumulative distribution function of density is expressed by the formula:

$$F(\rho) = \Phi_1 F_1(\rho) + \Phi_2 F_2(\rho) H(\rho - 2.0) \quad (26)$$

where:

- F_1 and F_2 – cumulative distribution functions, respectively, for the ranges of particle densities below and over 2.0 Mg/m^3 ,
- H – Heaviside's function (Byron et al. 1975),
- Φ_1 and Φ_2 – particle fractions of density below and over 2.0 Mg/m^3 in the sample ($\Phi_1 + \Phi_2 = 1$).

The graph of distribution function $F_2(\rho)$ is of a pictorial character in Fig. 1. It can be seen from the form of formula (26) that the density in the sample is a bi-modal distribution.

The dependence between density and mass content of ash in particle A was determined in order to find the distribution function of ash. The general form of this dependence is as follows:

$$A(\rho) = c_1 - \frac{c_2}{\rho} \quad (27)$$

where:

- c_1 and c_2 – constants, connected with the density of organic coal matter and the average density of mineral matter (Brožek 1995a).

The detailed form of this dependence is as follows:

$$A(\rho) = 1.54 - \frac{2}{\rho} \quad (28)$$

whereas density ρ is expressed in $[\text{Mg/m}^3]$ and A has fractional values.

Figure 2 presents the dependence of ash content on density inverse of fraction. The linear correlation coefficient is larger than 0.98. From formula (28) the dependence of density upon ash content in the particle is expressed by the formula:

$$\rho = \frac{2}{1.54 - A} \quad (29)$$

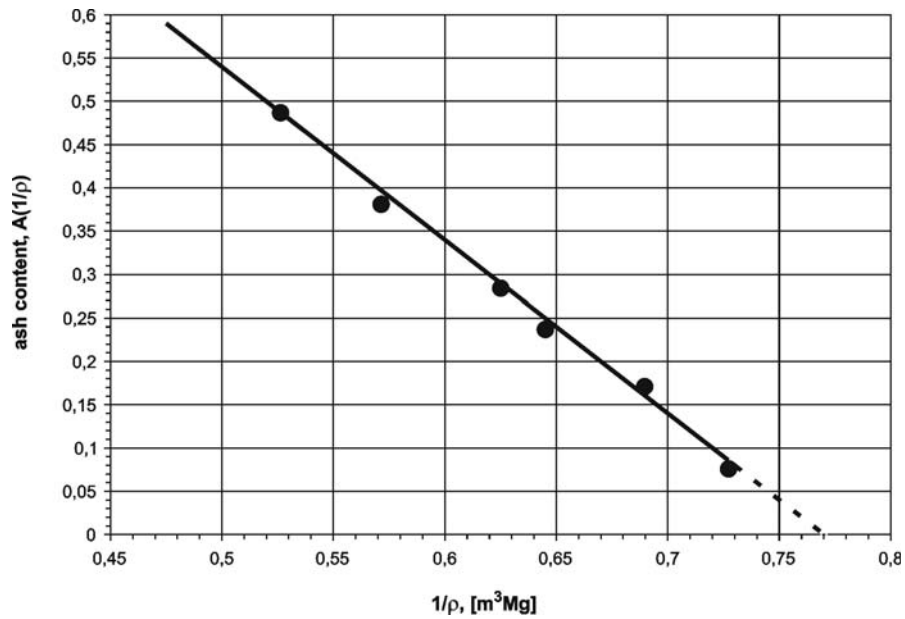


Fig. 2. Dependence of the ash content on the reciprocal of density fraction

Rys. 2. Zależność zawartości popiołu od odwrotności gęstości frakcji densymetrycznej

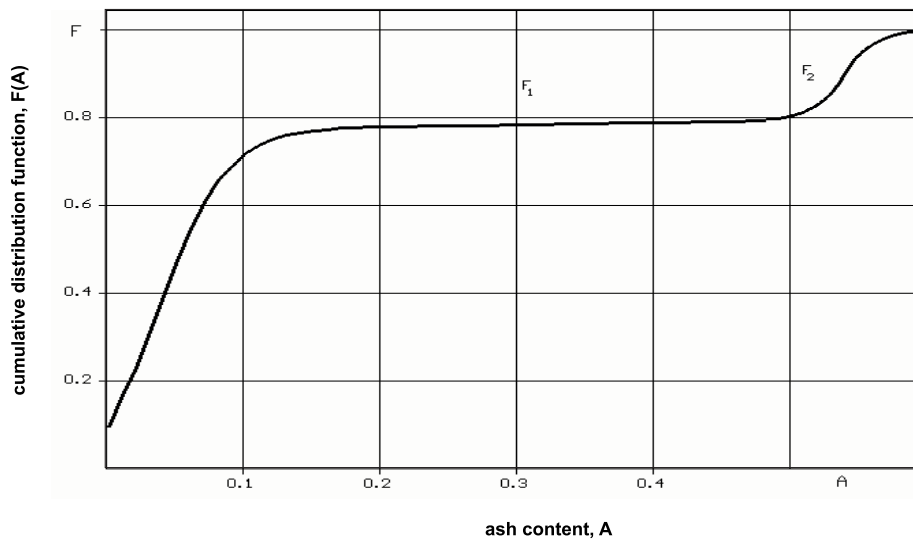


Fig. 3. Cumulative distribution function of the ash content for entire range of particle density

Rys. 3. Dystrybuanta rozkładu zawartości popiołu dla całego przedziału gęstości ziaren

After substituting expression (29) into formula (25) we may obtain the form of the cumulative distribution function of ash content for the range of particle density below 2.0 Mg/m^3 :

$$F_1(A) = 0.83 \left[0.94 I \left(2.2; 1 + \frac{39A}{1.54 - A} \right) \right] \quad (30)$$

While for the entire range of particle density:

$$F(A) = \Phi_1 F_1(A) + \Phi_2 F_2(A) H(A - 0.54) \quad (31)$$

The shape of the distribution function $F(A)$ is analogical to $F(\rho)$ and is shown in Fig. 3.

4.2. Kinetics of flotation of densimetric fractions

The investigations of flotation kinetics of respectively densimetric fractions were performed at butanol concentration of $2 \cdot 10^{-3} \text{ mola/dm}^3$ and the corresponding surface tension of solution of 68.5 [mN/m] . The curves of flotation kinetics were drawn according to the obtain results of cumulated recoveries of combustible and volatile matter in the froth product after flotation time t . Empirical dependences were approximated by the model equations of flotation kinetics of 1 st order (equation 3). The parameters of this equation R_∞ and $k = \lambda_0 + \mu_0$ were calculated applying the least square method for the given value R_∞ . The rate of compatibility of the model dependence with empirical courses was evaluated by means of

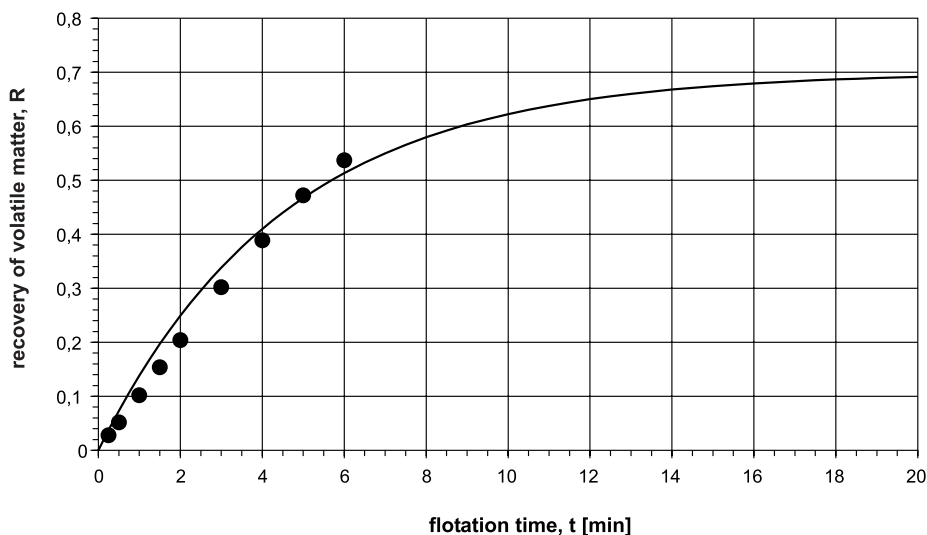


Fig. 4. Flotation kinetics of the fraction $<1.35 \text{ [Mg/m}^3]$, $R = 0,70(1 - e^{-0,22t})$, $\lambda_0 = 0,154 \text{ [1/min]}$

Rys. 4. Kinytyka flotacji, frakcja $<1,35 \text{ [Mg/m}^3]$, $R = 0,70(1 - e^{-0,22t})$, $\lambda_0 = 0,154 \text{ [1/min]}$

the curvilinear correlation index. The value R_{∞} were changed every 0.01 to reach the maximum value by the curvilinear correlation index. Figures 4–10 show model curves of flotation kinetics. Empirical values are marked with circles. In all cases the curvilinear correlation index is larger than 0.96.

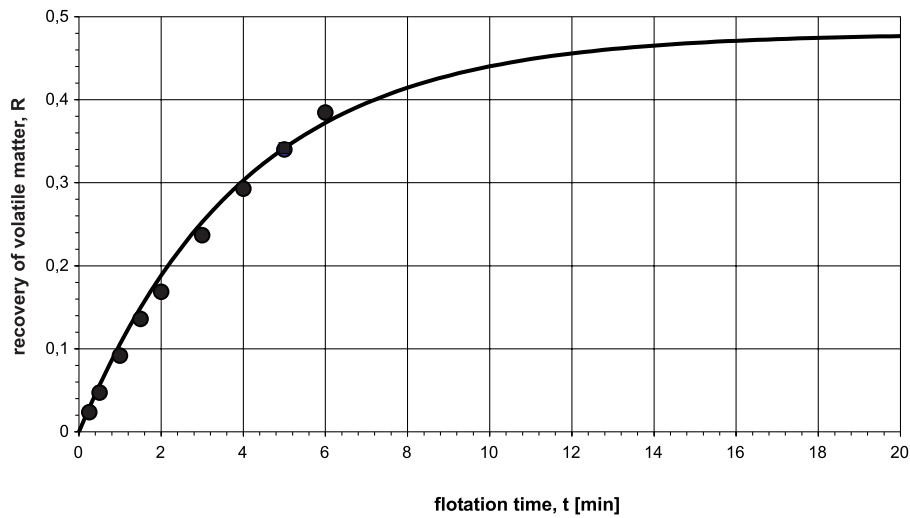


Fig. 5. Flotation kinetics of the fraction (1.35–1.5) [Mg/m³], $R = 0.48(1 - e^{-0.249t})$, $\lambda_0 = 0.12$ [1/min]

Rys. 5. Kinetyka flotacji, frakcja (1,35–1,5) [Mg/m³], $R = 0,48(1 - e^{-0,249t})$, $\lambda_0 = 0,12$ [1/min]

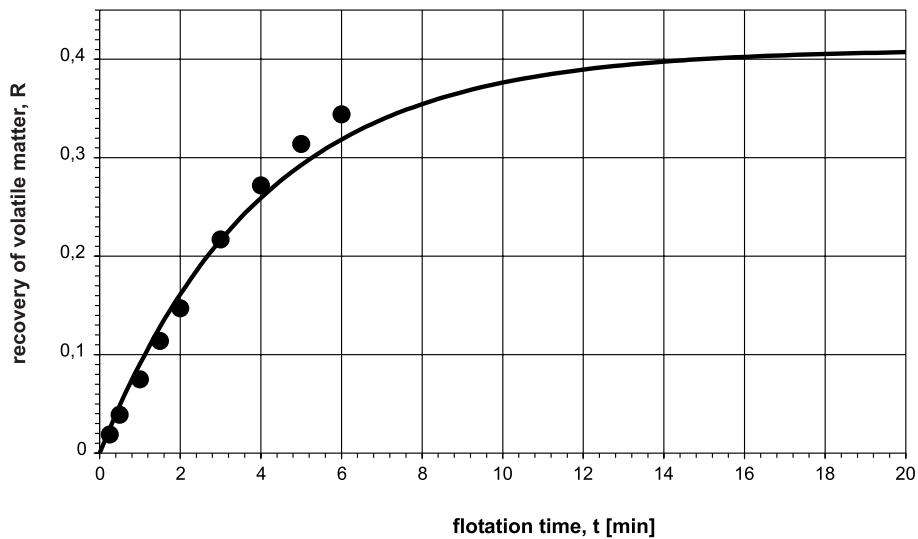


Fig. 6. Flotation kinetics of the fraction (1.5–1.7) [Mg/m³], $R = 0.41(1 - e^{-0.25t})$, $\lambda_0 = 0.102$ [1/min]

Rys. 6. Kinetyka flotacji, frakcja (1,5–1,7) [Mg/m³], $R = 0,41(1 - e^{-0,25t})$, $\lambda_0 = 0,102$ [1/min]

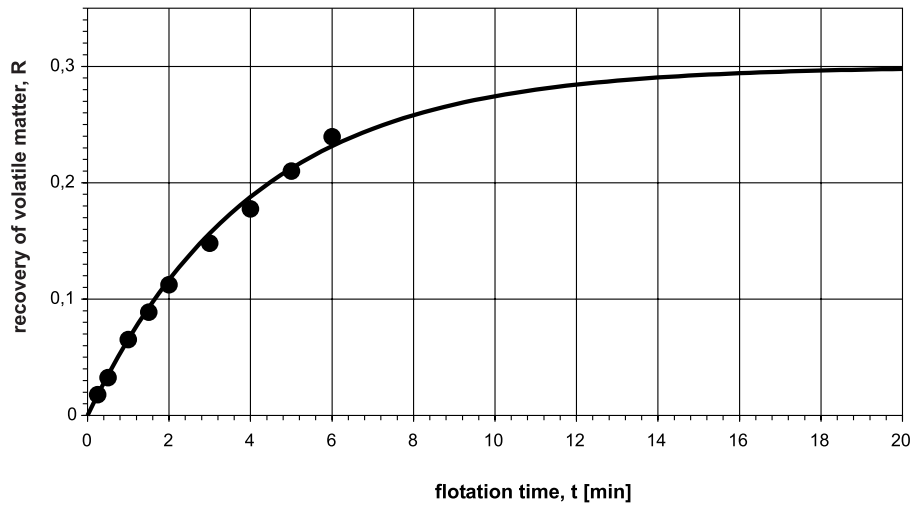


Fig. 7. Flotation kinetics of the fraction(1.7–1.8) [Mg/m³], $R = 0.30(1 - e^{-0.246t})$, $\lambda_0 = 0.074$ [1/min]

Rys. 7. Kinetyka flotacji, frakcja (1,7–1,8) [Mg/m³], $R = 0,30(1 - e^{-0.246t})$, $\lambda_0 = 0,074$ [1/min]

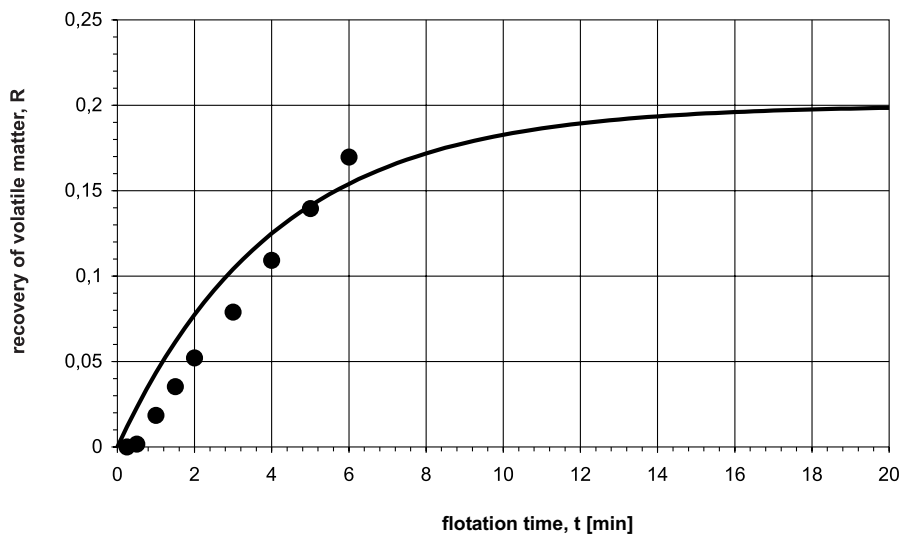


Fig. 8. Flotation kinetics of the fraction (1.8–2.0) [Mg/m³], $R = 0.20(1 - e^{-0.245t})$, $\lambda_0 = 0.079$ [1/min]

Rys. 8. Kinetyka flotacji, frakcja (1,8–2,0) [Mg/m³], $R = 0,20(1 - e^{-0.245t})$, $\lambda_0 = 0,079$ [1/min]

Adhesion rate constant was calculated according to model dependence $R(t)$, and to the formula:

$$\lambda_0 = \lim_{t \rightarrow \infty} \frac{dR(t)}{dt} = kR_\infty \quad (32)$$

The model dependence $R(t)$ and the value of adhesion rate constant λ_0 calculated for it, are given with every graph. Fig. 10 shows the dependence of adhesion rate constant on ash content in the sample subjected to flotation. This dependence, according to formula (3), is as follows:

$$\lambda_{0(A)} = 0.15e^{-2A} \quad (33)$$

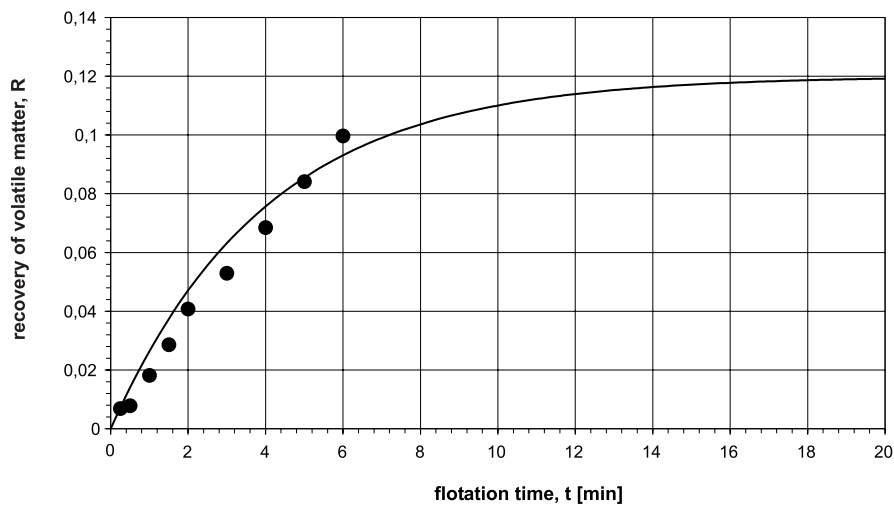


Fig. 9. Flotation kinetics of the fraction >2.0 [Mg/m^3], $R = 0.12(1 - e^{-0.249t})$, $\lambda_0 = 0.03$ [1/min]

Rys. 9. Kinetyka flotacji, frakcja $>2,0$ [Mg/m^3], $R = 0,12(1 - e^{-0,249t})$, $\lambda_0 = 0,03$ [1/min]

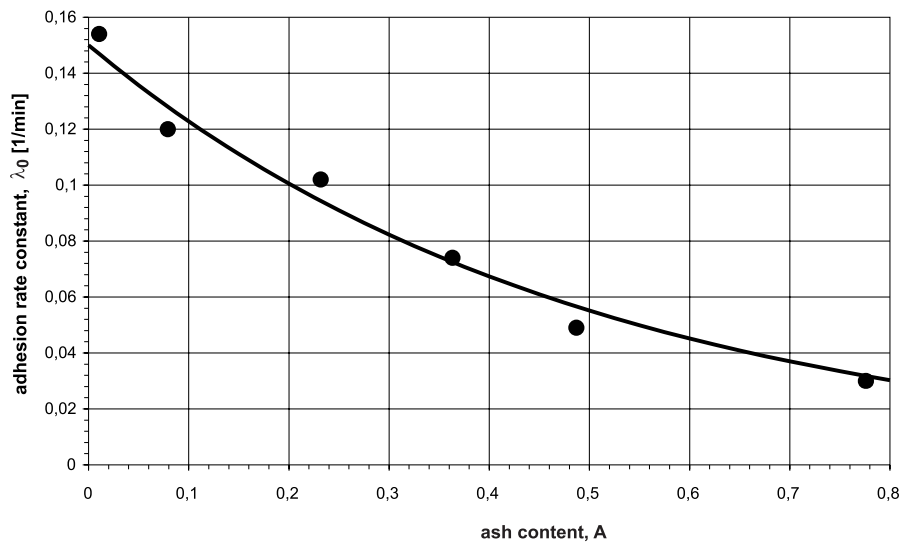


Fig. 10. Adhesion rate constant as a function of the ash content $\lambda_0 = 0.15e^{-2A}$

Rys. 10. Stała prędkości adhezji w funkcji zawartości popiołu w próbce; $\lambda_0 = 0,15e^{-2A}$

The curvilinear correlation index is 0.9999022 which proves a very good compatibility of the proposed model dependence with experimental values.

4.3. Cumulative distribution function of adhesion rate constant

The ash content, according to formula (33), is:

$$A = \frac{1}{2} \ln \frac{0.15}{\lambda_o} \quad (34)$$

Substituting the above dependence into formula (30) we obtain the cumulative distribution function rate constant for the particles density below 2.0 Mg/m³:

$$F_1(\lambda_o) = 1 - 0.83 \left[0.87I \left(2.2; \frac{-38 \ln \lambda_o - 69}{\ln \lambda_o + 5} \right) \right] \quad (35)$$

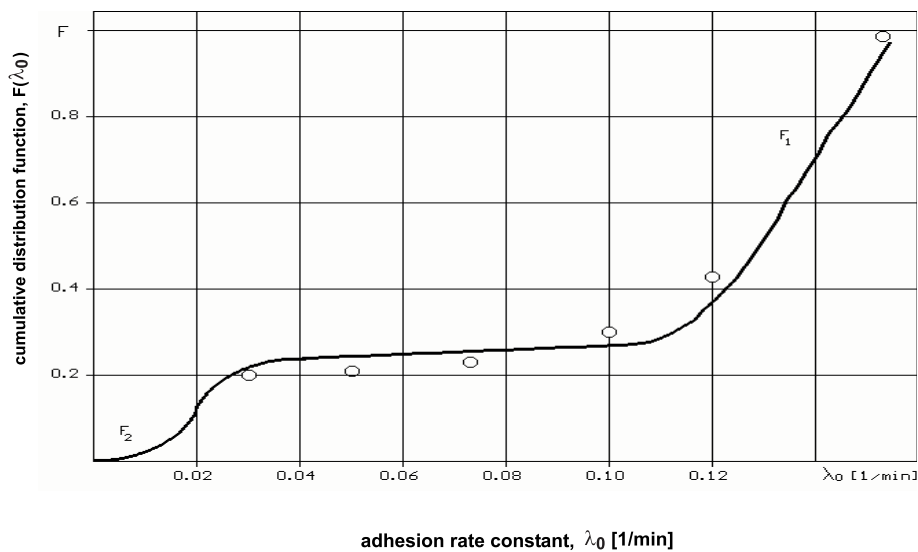


Fig. 11. Cumulative distribution function of adhesion rate constant

Rys. 11. Dystrybuanta rozkładu stałej prędkości adhezji

The graph of function (35) is presented in Fig. 11 together with experimental values. The curvilinear correlation index is larger than 0.95. Analogically as in the case of distribution function of density and mineral matter content, the distribution function of adhesion rate constant for the entire range of variability λ_o is expressed by the formula:

$$F(\lambda_o) = \Phi_1 F_1(\lambda_o) + \Phi_2 F_2(\lambda_o) H(\lambda_o - 0.04) \quad (36)$$

Analogically, the frequency function is as follows:

$$f(\lambda_o) = \Phi_1 f_1(\lambda_o) + \Phi_2 f_2(\lambda_o) \quad (37)$$

Figure 12 shows the frequency function of adhesion rate constant, and the first part of dependence $f_2(\lambda_o)$ is of pictorial character. It is, therefore, the bi-modal distribution.

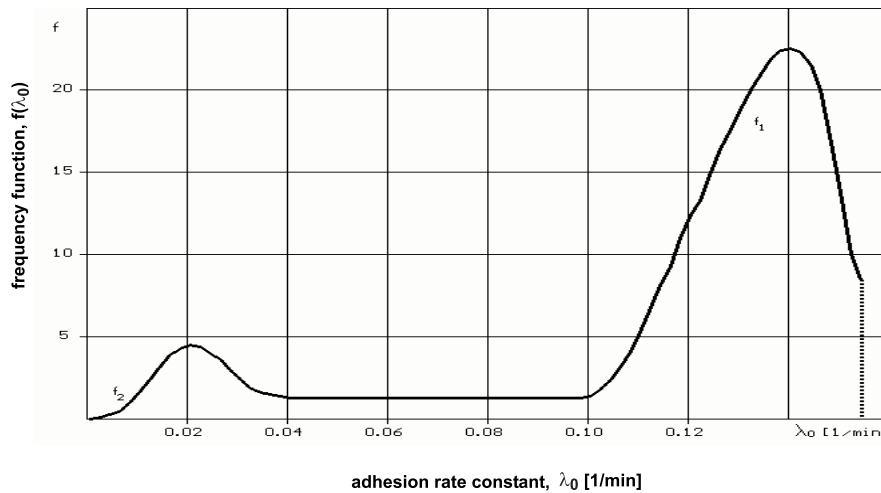


Fig. 12. Frequency function of the adhesion rate constant

Rys. 12. Funkcja gęstości rozkładu stałej prędkości adhezji

The bi-modal distribution of flotation rate constant was proposed by Kalinowski and Kaula (1994). Such a type of distribution results from the character of distribution of the mineral matter content in coal. It can be, therefore, only grounded by combining the flotation properties of particles with the mineral matter content.

Conclusions

1. The flotation, as every separation process, is a mass process. Bubble mineralization is the result of two random events; a collision of the particle with the air bubble and adhesion of the particle to the bubble surface. Because of this, in the macroscopic approach, the process of flotation can be treated as a stochastic process.
2. Due to the fact that bubble mineralization in the flotation chamber occurs in turbulent conditions, despite the process of adhesion of particles to the bubble surface there is a reverse process of lower intensity, namely the detachment of the particles from bubble

surfaces. Therefore, to describe the kinetics of flotation, the stochastic model of birth and death was applied in this work, characterized by two parameters – the permanent adhesion rate constant and the detachment rate constant.

3. The permanent adhesion rate constant for the tested coal (type 33 according to the Polish classification) decreases exponentially with the increase of ash content. The curvilinear correlation index for this dependence is larger than 0.99. Thus a theoretical justification for applying this type of dependences for other coal types should be sought.
4. Basing on the assumption of existence of a relation between surface and volume properties of the particle, the distribution of adhesion rate constant was determined on the grounds of a physical model of the particle. This distribution is correlated with the distribution of the mineral matter content. Only on the grounds of this model it is possible to justify the application of the bi-modal distribution in the investigations of flotation kinetics of non-homogeneous materials from the point of view of flotation properties.

The paper has been supported by a grant of the National Research Committee no 4 T12A 035 30

REFERENCES

- Bailey N.T.J., 1964 – The Elements of Stochastic Process with Application to the Natural Sciences. John Wiley&Sons, New York, London, Sydney.
- Brożek M., 1995a – Densimetric and magnetic characteristics of crushed material. Archives of Mining Sciences vol. 40, p. 63–82.
- Brożek M., 1995b – The distribution of selected physical properties in the crushed material. Archives of Mining Sciences vol. 40, p. 83–100.
- Brożek M., 1995c – The distribution of magnetic susceptibility in crushed ores. Magnetic and Electrical Separation vol. 6, p. 213–228.
- Byron F.W., Fuller R.W., 1975 – Matematyka w fizyce klasycznej i kwantowej. Warszawa, PWN.
- Cottrell A.H., 1964 – The mechanical properties of matter. New York, J. Wiley & Sons.
- Firkowicz S., 1970 – Statystyczne badanie wyrobów. Warszawa, WNT.
- Fisz M., 1967 – Rachunek prawdopodobieństwa i statystyka matematyczna. Warszawa, PWN.
- Geidel T., 1985 – Probability of attachment between mineral grains and air bubbles and its relation to flotation kinetics. Aufbereitungs Technik vol. 26, p. 287–294.
- Gradstein I.S., Ryzik I.M., 1971 – Tablicy integralow, summ, riadov i proizvedenij. Moskva, Izd. Nauka.
- Harris C.C., Cuadros-Paz A., 1978 – Species interaction in flotation: a laboratory – scale semi-batch study. International Journal of Mineral Processing vol. 5, p. 267–283.
- Inoue T., Imazumi T., 1968 – Some aspects of flotation kinetics. Proc. VIII IMPC vol. II, p. 368–398, Leningrad.
- Kalinowski K., Kaula R., 1994 – A verification method of coal flotation kinetic models. Proc. XII ICPC vol. II, p. 1153–1158, Cracow.
- Krzan M., Małysa K., 2002 – Profiles of local velocities of bubbles in *n*-butanol, *n*-hexanol, *n*-nonanol solution. Colloids and Surfaces. A: Physicochemical and Engineering Aspects vol. 207, p. 279–291.
- Mehrotra S.P., Kapur P.C., 1974 – The effects of aeration rate, particle size and pulp density on the flotation rate distribution. Powder Technology vol. 9, p. 213–219.
- Pagurova V.I., 1963 – Tablicy nepolnoj gamma-funkcji. Vycislitelnyj Centr AN SSSR, Moskwa.
- Pogorelyj A.D., 1961 – O flotacionnoj charakteristike promyslennoj pulpy. Izv. VUZ Cvetnaja Metallurgija no 5, s. 59–68.

- Sablík J., 1998 – Flotacja węgla kamiennych. Główny Instytut Górnictwa. Katowice.
- Schulze H.J., 1992a – Interface actions in mineral processes. *Aufbereitungs Technik* vol. 33, p. 434–443.
- Schulze H.J., 1992b – Interface actions in mineral processes. *Aufbereitungs Technik* vol. 33, p. 691–697.
- Stachurski J., 1972 – Matematyczny model procesu bezpianowej flotacji jonowej. *Zeszyty Naukowe AGH* nr 381, s. 55–64.
- Tille P., Panou Z., 1968 – Some consideration on flotation kinetics. *Proc.VIII IMPC* vol. II, p. 487–499, Leningrad.
- Woodburn E.T., Loveday B.K., 1965 – The effect of variables residence time on the performance of flotation system, *Journal of. South African Institute of Mining and Metallurgy* vol. 65, no 12, p. 612–628.
- Xu Changlian, 1985. Kinetic models for batch and continuous flotation in a flotation column. *XV IMPC* vol. III, p. 16–27, Cannes.

**ZWIĄZEK MIĘDZY DYSPERSYJNYM MODELEM ZIARNA A ROZKŁADEM STAŁEJ PRĘDKOŚCI ADHEZJI TRWAŁEJ
W PROCESIE FLOTACJI WĘGLA**

Słowa kluczowe

Flotacja węgla, kinetyka flotacji, model dyspersyjny ziarna, proces narodzin i giniecia, stała prędkości adhezji, stała prędkości odrywania, rozkład stałej prędkości adhezji

Streszczenie

Flotacja jest procesem wzbogacania mającym na celu zróżnicowanie zawartości składnika użytecznego w produktach rozdziału, a więc zróżnicowanie właściwości objętościowej ziarna. Wykorzystuje się do tego celu zróżnicowanie właściwości powierzchniowych ziarna. Nasuwa się więc przypuszczenie, że powinien istnieć związek pomiędzy właściwościami powierzchniowymi ziarna lub wielkościami uzależnionymi od właściwości powierzchniowych a właściwościami objętościowymi.

W praktyce wzbogacania flotacyjnego przyjęło się określać rozkład właściwości flotacyjnych w próbce poprzez rozkład stałej prędkości flotacji w równaniu kinetyki I rzędu. Jeżeli flotację rozpatruje się jako proces stochastyczny narodzin i giniecia, wtedy rozkład właściwości flotacyjnych charakteryzuje się poprzez rozkład stałej prędkości adhezji trwałej. Model narodzin i giniecia pozwala na ilościowe określenie dwóch subprocesów zachodzących w komorze flotacyjnej, a mianowicie procesu adhezji oraz procesu odrywania już przyczepionych ziaren od pęcherzyków powietrza na skutek turbulencji ośrodka.

W artykule przedstawiono omówienie modelu narodzin i giniecia, wyspecyfikowano funkcję intensywności procesu adhezji i odrywania (wyrażenie 13), podano równanie różniczkowe modelu (wyrażenie 14) oraz jego rozwiązanie (wyrażenie 18), w którym w sposób jawny występuje stała procesu adhezji λ_o i stała odrywania μ_o . Z dopasowania empirycznej zależności uzysku składnika flotowanego od czasu flotacji do zależności modelowej otrzymuje się wartość stałej prędkości adhezji trwałej λ_o .

Na podstawie dyspersyjnego modelu ziarna podano wyrażenie na dystrybuantę rozkładu zawartości fazy rozproszonej (wyrażenia 21 i 22), a następnie z empirycznego związku pomiędzy stałą prędkości adhezji trwałej a zawartością popiołu (wyrażenia 3 i 4) uzyskano ogólne wyrażenie na dystrybuantę rozkładu stałej prędkości adhezji w postaci niepełnej funkcji gamma (wyrażenie 23).

Badania laboratoryjne wykonano na próbce węgla (typ 33) o uziarnieniu 0,2–0,315 mm. Na podstawie analizy densymetrycznej uzyskano dystrybuantę rozkładu gęstości (wyrażenie 25 i rys. 1), a za pomocą zależności pomiędzy zawartością popiołu a gęstością (wyrażenia 28 i 29 oraz rys. 2) dystrybuantę rozkładu zawartości popiołu (wyrażenie 30 i rys. 3).

Badania kinetyki flotacji poszczególnych frakcji densymetrycznych wykonano z użyciem butanolu o stężeniu $2 \cdot 10^{-3}$ mola/dm³. Krzywe kinetyki flotacji wraz z równaniami modelowymi oraz wartościami stałej prędkości adhezji trwałej przedstawiają rysunki 4–9. Zależność empiryczna stałej prędkości adhezji od zawartości popiołu jest określona przez wyrażenie (33) i rys. 10. Podstawienie zależności odwrotnej do zależności (33) do wyrażenia

na dystrybuantę rozkładu zawartości popiołu (wyrażenie 30) daje ostatecznie empiryczną postać dystrybuanty rozkładu stałej prędkości adhezji trwałej (wyrażenie 35 i rys. 11) lub po zróżniczkowaniu funkcję gęstości rozkładu przedstawioną na rysunku 12.

We wnioskach stwierdzono, że można wyznaczyć rozkład stałej prędkości adhezji trwałej opierając się na dyspersyjnym modelu ziarna i związku empirycznym pomiędzy stałą prędkości adhezji a zawartością popiołu. Należy w związku z tym poszukiwać teoretycznego uzasadnienia dla zastosowania tego rodzaju zależności (wzór 3) dla innych typów węgla.

**THE RELATION BETWEEN THE DISPERSIVE MODEL OF THE PARTICLE AND THE DISTRIBUTION
OF PERMANENT ADHESION RATE CONSTANT IN THE COAL FLOTATION PROCESS**

Key words

Flotation kinetics, flotation kinetics, physical model of particle, stochastic process of birth and death, adhesion rate constant, detachment rate constant, distribution of adhesion rate constant

Abstract

Flotation as a random process in which the random variable representing the number of particles raised to the froth layer depends on time, is a stochastic process. As a result of turbulence of medium in the flotation chamber, despite the process of adhesion to bubbles, there is a reverse process of lower intensity, i.e. the process of detachment the particles from the air bubbles. Such a situation is described by the stochastic process of birth and death.

The paper presents the assumption and differential equations of the pure death process and process of birth and death.

Flotation is the process of enrichment which consists in differentiating the useful component (volume property) in the separation products. Flotation leads to the differentiation of the volume property by means of applying the differentiation of surface properties. Since there is a correlation between these properties. At assumption that mineral matter is evenly distributed in the volume of coal particle, the surface content of mineral matter is proportional to the volume content. Amount of mineral matter on the coal surface determines hydrophobic properties of particle. The authors determined the distribution of adhesion rate constant in relation with the content of the useful component, applying the dispersive model of a particle. The content of the mineral matter is directly connected with the volume physical property, represented by particle density.

The paper present distribution functions of density and adhesion rate constant in the sample. Also the relation between adhesion rate constant and ash content for narrow density fractions has been revealed.