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The study of decision making tools for equipment selection in mining engineering operations

Introduction

Decision making may be characterized as a process of choosing or selecting 'sufficiently good' alternative(s), to attain, from a set of alternatives, to attain a goal or goals. Much decision making involves uncertainty. Hence, one of the most important aspects for a useful decision aid is to provide the ability to handle imprecise and vague information, such as 'large' profits, 'fast' speed and 'cheap' price. A decision model should cover process for identifying, measuring and combining criteria and alternatives to build a conceptual model for decisions and evaluations in fuzzy environments. Mine planning engineers often use of their intuition and experiences in decision making. Mostly linguistic variables (the weather is raining, soil is wet, etc.) become in question and decision-makers may not know how these variables are computed. Since the advent of the modern methods like as the fuzzy set theory and the analytic hierarchy process (AHP), these uncertainties are easily evaluated in decision making process. By the development of computer technology and programming of colloquial language with expert systems have considerably reduced decision makers' burden.

1. Decision making tools

Multiple attribute decision making (MADM) deals with the problem of choosing an alternative from a set of alternatives which are characterised in terms of their attributes.

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Usually MADM consists of a single goal, but this may be of two different type. The first is where the goal is to select an alternative from a set of scored ones based on the values and importance of the attributes of each alternative. The second type of goal is to classify alternatives, using a kind of role model or similar cases. MADM is a qualitative approach due to the existence of criteria subjectivity. Both type of goals require information about the preferences among the instances of an attribute and the preferences across the existing attributes (Riberio 1993). The assessment of these preferences is either provided directly by the decision maker or based on past choices. The general formalisation is:

Let $A_1, A_2, \ldots, A_m$ be a set of alternatives to be assessed by criteria $C_1, C_2, \ldots, C_n$.

Let R_{ij} be the numerical rating of alternative A_i for criteria C_j.

Then the general decision function is:

$D(A_i) = (R_{i1} \cdot R_{i2} \cdot \ldots \cdot R_{ij})$ for $j = 1, 2, \ldots n$ and $\cdot$ represents the aggregation.

Further, the decision maker might express or define a ranking for the criteria as importance/weights. There are many forms for expressing these importance, but the most common are : (a) utility preference functions; (b) the analytical hierarchy process (Saaty 1978a, b; Yager 1978) and (c) fuzzy version of the classical linear weighted average (Baas, Kwakernaak 1977; Baldwin, Guilg 1979). In addition, even for a fuzzy decision, the criteria could be fuzzy or crisp. The aim of the MADM is to obtain the best alternative, that is the one with the highest degree of satisfaction for all the relevant attributes or goals. In order to obtain the best alternative a ranking process is required. If the rating for alternative Ak is crisp, there is no problem and the best alternative is the one with the highest support. When the rating is itself a fuzzy set, a more sophisticated ranking procedure is required.

1.1. The analytic hierarchy process

This method has been developed by Saaty (1980). The AHP structures the decision problem in levels which correspond to one's understanding of the situation: goals, criterion, sub-criterion, and alternatives. By breaking the problem into levels, the decision-maker can focus on smaller sets of decisions. The AHP is based on 4 main axioms:

(1) Given any two alternatives (or sub-criterion), the decision-maker is able to provide a pairwise comparison of these alternatives under any criterion on a ratio scale which is reciprocal.

(2) When comparing any two alternatives, the decision-maker never judges one to be infinitely better than another under any criterion.

(3) One can formulate the decision problem as a hierarchy.

(4) All Criterion and alternatives which impact a decision-problem are represented in the hierarchy.

The above axioms describe the two basic tasks in the AHP: formulating and solving problem as a hierarchy, and eliciting judgements in the form of pairwise comparisons. The elicitation of priorities for a given set of alternatives under a given criterion involves the completion of a nxn matrix, where n is the number of alternatives under consideration. Hovewer, since the comparisons are assumed to be reciprocal, one needs to answer only n(n-1)/2 of the comparisons. Saaty proposed an eigenvector approach for the estimation of the weights from a matrix of pairwise comparisons. The eigenvector also has an intuitive interpretation in that it is an averaging of all possible ways of thinking about a given set of alternatives. After estimating the weights, the decision-maker is also provided with a measure of the inconsistency of the given pairwise comparisons. It is important to note that the AHP does not require decision-makers to be consistent but, rather, provides a measure of inconsistency as well as a method to reduce this measure if it is deemed to be too high. After genereting a set of weights for each alternative under any criterion, the overall priority of the alternatives is computed by means od a linear, additive function (Munda 1995; Albayrak C., Toraman, Albayrak E. 1997).

The method measures relative fuzziness by structuring the criterion and objectives of a system, hierarchicaly in a multiple attribute framework. In order to rate the alternatives Saaty (1978a, b) uses a hierarchical pair-wise comparison between attributes and/or objectives and then solves them with eigenvectors of the reciprocal matrices.

1.2. Fuzzy set theory

Researchers and practitioners of equipment and method selection face diverse operational issues such as the complexity of interactive influences, inaccuracy of measures, uncertainty of environmental forces, and subjectivity of the decision making process. Acquiring the information necessary for equipment and method selection is elaborate, to say the least, and once obtained is liable to be ambiguous, inconsistent, incomplete, or deficient in quality. In addition, decision makers must often apply rules of thumb or incorporate their personal intuition and judgment when deriving performance measures based on indefinite linguistic concepts, e.g. ‘high’, ‘low’, ‘strong’, ‘weak’, ‘stable’, and ‘deteriorating’. For example, if haulage level is low and then truck haulage can be used (the opportunity will be high). If coal seam is about 2.0 meters in thickness and has weak hanging wall condition then longwall method with filling can be good choice in alternatives. This terminology is a natural phenomenon caused by imperfectly defined problem attributes.

Fuzzy sets (Zadeh 1975) have vague boundaries and are therefore well suited for discussing such concepts as linguistic terms (such as very or somewhat) or natural phenomena (temperatures). Fuzzy set theory has developed as an alternative to ordinary (crisp) set theory and is used to describe fuzzy sets. To clear the difference between these two sets, let explain with an example. Supposed that a set *K* has various cycle times of one shovel loading same size trucks between 20 and 28 seconds. An optimum loading cycle time is considered to be 24 to 25 seconds in this mine site. *K* set is firstly evaluated by crisp and subsequently by fuzzy set.

Figure 1 (a) shows crisp set of cycle time in the 24 to 25 seconds range. In this set, 24 seconds is 100 percent a member while 23 seconds is not in the set at all; there is no in-between. The boundaries are definite and a particular loading cycle time is either in the set

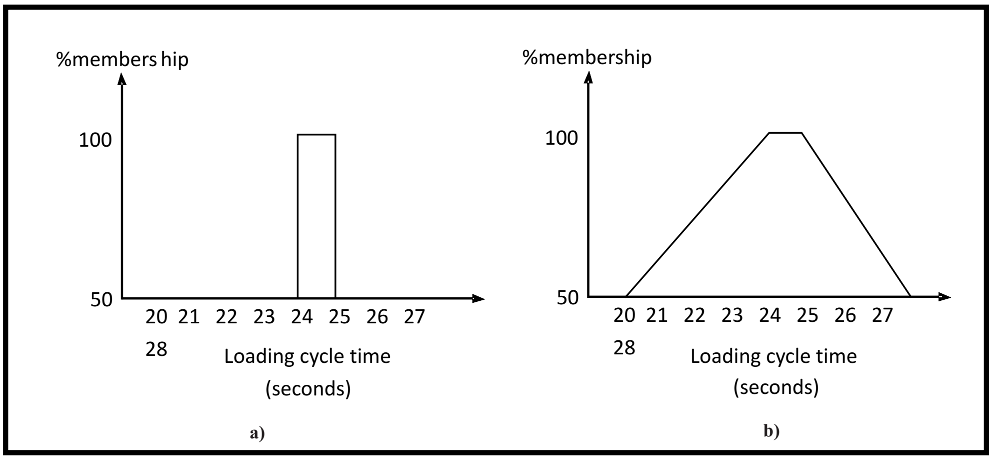

Fig. 1. Crisp (a) and fuzzy set (b)

Rys. 1. Zbiór normalny (a) i rozmyty (b)

or not, is either 24 to 25 seconds, or is not. In contrast, a fuzzy set does not have clear boundaries. Membership in a fuzzy set is a matter of degree. Figure 1 (b) shows 25 seconds is 100 percent a member of the set of loading cycle time, whereas 22 seconds is only 50 percent a member of the set.

Additionally, the nature of fuzzy sets allows something to be a member in more than one fuzzy set. For example, a 3-year-old haulage truck might be 20 percent a member of the set of young trucks and 45 percent a member of the set of middle-aged trucks.

Driving the set membership function for a fuzzy set is through the use in fuzzy logic or fuzzy decision making. The problem of constructing meaningful membership functions has a lot of additional research work that will have to be done on it to achieve full satisfaction. There are a number of empirical ways to establish membership functions for fuzzy sets. Measuring of these is beyond the scope of this article. However, for more information see Li et al. (1995); Klir et al. (1995). There are many methods of decision making. The focus of this paper is on Yager's (1981) method that is general enough to deal both with multiple objective and multiple attribute problems. Concentrating on multiple attribute decision making problems, only a single objective is considered, that a selecting the 'best' from a set of alternative. All other objectives are considered criteria. The method assumes the max-min principle approach. Formally, let $A = \{A_1, A_2, .., A_n\}$ be the set of alternatives, $C = \{C_1, C_2,.., C_m\}$ be the set of criteria which can be given as fuzzy sets in the space of alternatives, and G the goal, which can also be given by a fuzzy set. Hence, the fuzzy set decision is the intersection of all criteria (and/or goals):

$\mu_D(A) = \min(\mu_G(A), \mu_{C1}(A), \mu_{C2}(A), \ldots, \mu_{CM}(A))$ for all $A_i \in A$ and the optimal decision,
$\mu_D(A^*) = \max_A \mu_D(A)$ where A^* is the optimal decision.

The main difference is that the importance of criteria is represented as exponential scalars. This is based on the idea of linguistic hedges (Zadeh 1975). The rationale behind using weights (importance) as exponents is that the higher the importance of criteria the larger should be exponent because of the minimum rule. Conversely, the less important, the smaller the weight. This seems intuitive. Formally

$$\mu_D(A) = \min\left((\mu_G(A))^{\alpha 1}\ (\mu_{C1}(A))^{\alpha 2}, (\mu_{C2}(A))^{\alpha 3}, \ldots, (\mu_{CM}(A))^{\alpha m}\right) \quad \text{for } \alpha > 0$$

Consider the problem of selecting a site from the set {*A, B, C*} for a new in-pit crusher in a quarry, with the goal, *G*, of spending the minimum investment possible and for criteria evaluation to be located near the pit and the processing plant, respectively *C*1 and *C*2. The judgment scale used is 1-Equally important, 3-weakly more important, 5-strongly more important, 7-demonstrably more important and 9-absolutely more important. The values between (2, 4, 6, 8) show compromise judgments. Yager suggests the use of Saaty's (1978b) method for pair-wise comparison of the criteria (attributes). A pair-wise comparison of attributes (criteria) could improve and facilitate the assessment of criteria importance. Saaty developed a procedure for obtaining a ratio scale for the elements compared. To obtain the importance the decision-maker is asked to judge the criteria in pair-wise comparisons and the values assigned are $\boldsymbol{w_{ij} = 1/w_{ij}}$. Having obtained the judgments, the mxm matrix *B* is constructed so that: (a) $b_{ii} = 1$; (b) $b_{ij} = w_{ij}$; (c) $b_{ji} = b_{ij}$. To sum up, Yager suggests that the resulting eigenvector should be used to express the decision maker's empirical estimate of importance (the reciprocal matrix in which the values are given by the decision maker) for each criteria in the decision and criteria 1 and 2, respectively $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$, are three times as important as *G*, and the pair-wise comparison reciprocal matrix is:

	G	C_1	C_2
G	1	1/ 3	1/ 3
C_1	3	1	1
C_2	3	1	1

Hence, the eigenvalues of the reciprocal matrix are $\lambda = [0, 3, 0]$ and therefore $\lambda_{max} = 3$. All values except one are zero (Saaty 1978b). The weights of the criteria are finally achieved in the eigenvector of the matrix, eigenvector = {0.299, 0.688, 0.688} with λ_{max}. The eigenvector corresponds to the weights to be associated with the memberships of each attribute/feature/goal. Thus, the exponential weighting is $\alpha_1 = 0.299$, $\alpha_2 = 0.688$, $\alpha_3 = 0.688$ and the final decision (membership decision function) about the site location is given as follows:

$$D(A) = \min(G^{0.299},\ C_1^{0.688},\ C_2^{0.688})$$
$$G = [0.5/A_1,\ 0.8/A_2,\ 0.3/A_3]^{0.299} = [0.81/A_1,\ 0.94/A_2,\ 0.70/A_3]$$

C_1= $[0.7/A_1, 0.9/A_2, 0.5/A_3]^{0.688}$ = $[0.78/A_1, 0.93/A_2, 0.62/A_3]$

C_2= $[0.4/A_1, 0.2/A_2, 0.9/A_3]^{0.688}$ = $[0.53/A_1, 0.33/A_2, 0.93/A_3]$

$D(A) = (0.53/A_1, 0.33/A_2, 0.62/A_3)$

And the optimal solution (Riberio 1993), corresponding to the maximum membership 0.62, is

$A_3(\mu_D(A^*) = 0.62/A_3)$.

2. Applications of the AHP and fuzzy set theory in mining

In spite of many advantages of the approach, only few applications of AHP model to mining industry problems have been reported in the literature. Some of these applications are briefly reviewed here:

Bandopadhyay et al. (1986, 1987a) developed fuzzy algorithm for selection of post-mining uses of land and for decision making in mining engineering. Bandopadhyay (1987b) indicated partial ranking of primary stripping equipment in surface mine planning and fuzzy algorithm. It deals with the process of ranking alternatives after determining their rating. Determining the optimal decision alternatives, when the results are crisp, is straight-forward (just select the alternative with the highest support). It also considers that the supports for each alternative are themselves fuzzy sets. Therefore, in order to select the 'best' alternative more sophisticated methods of comparison are needed. Gershon et al. (1993) studied mining method selection: a decision support system integrating multi-attribute utility theory and expert systems. Herzog et al. (1996) indicated ranking of optimum beneficiation methods via the analytical hierarchy process. This process measures relative 'fuzziness' by structuring the Criterion and objectives of a system, hierarchically, in a multiple attribute framework. Bascetin et al. (1999a) handled the study of a fuzzy set theory for the selection of an optimum coal transportation system from pit to the power plant. This project has comprised variety of criterion related to the coal transportation systems. Kesimal et al. (2002) handled application of fuzzy multiple attribute decision making in mining operations. In this project, the AHP model was used together with fuzzy set theory for equipment selection. Bascetin et al. (1999b) handled Application of Fuzzy Logic in Mining. It deals with the AHP and Fuzzy Logic Applications in Multiple Attribute Decision Environment. Karadogan et al. (2001) studied underground mining method selection using analytic hierarchy process within fuzzy algorithm. Bascetin (1999) studied optimal equipment selection in open pit mines using the AHP and fuzzy set theory. Bascetin (2007) applied the AHP to the problem of optimal environmental reclamation for open pit mines. Bascetin et al. (2006) were developed a new software for equipment selection using fuzzy logic in the paper. Bascetin (2004) used only the AHP for equipment selection without fuzzy logic.

3. Case study

In this case study it has been carried out some researches on loading-hauling systems for coal production to be established in an open pit coal mine located Orhaneli, western part of Turkey. The coal mine is situated about 65 km north of Bursa located west of Turkey. The mine has been in continuous operation since 1979. Currently the mine supplies Orhaneli power plant unit (1 · 210 MW) and some for domestic use only. The lowest calorific value lignite (2704 kcal/kg) will be mined for electricity generation. In this case, the overall measurements of the mine should be designed again in terms of transporting system, equipment fleet, etc. Technical parameters of working site, which affect the systems, have been searched thoroughly and summarized below in detail (Bascetin 2004).

The present extent of the open pit is 1200 m long by 400 m wide and a total of 75 m of overburden being removed in three 15 m high benches and an average thickness of 7 m coal being mined at one bench only. The last 25 m. of overburden from surface is mined using dragline. The face inclination on individual benches is 75° while overall pit slopes are 45°. The average temperatures are varying between 30°C and minus 6°C. The average temperatures yearly is 14°C.

The mine will be worked over 18 years at the rate of one shift (12 h/d) per day, seven days a week for 300 days per year, the scheduled operating time being 3600 h/year. The average coal production is planned to be 1.300.000 t/year, (100.000 t/year upper 3.500 kcal/kg for domestic use, 1.200.000 t/year 2700 kcal/kg for power plant) which implies an average annual overburden removal of 15.000.000 m^3 -i.e. the economic mine life is based on the first tenth year 11.43 m^3/ton and between tenth and eighteenth year 11 m^3/ton stripping ratio.

Three drilling units are employed for overburden, four being 9 inch DM50. Coal bench has an easy diggibility. Sometimes blasting is applied for getting big size coal. Two rope shovels (Marion 191 MII) fitted with 15.3 m^3 buckets, four PH 1900 AL shovels with 7.64 m^3 buckets, one dragline (1260-W Bucyrus-Erie) with 25 m^3 bucket in waste and one as front shovel with 7.64 m^3 buckets in coal are used for loading. A fleet of totally forty-four off-highway trucks undertakes haulage. Twentey-seven (Caterpiller 777-77 ton), thirteen (Komatsu 785-2, 77 ton), four (Komatsu 785-2, 50 ton) are equipped as carrying waste and six (Komatsu HD 465-3) as coal trucks with 50 tonnes capacity. Average haul distances are 2.500 m with coal, 2.000 m with waste.

Supporting equipment in the mine includes five Komatsu D355A bulldozer of 410 hp; three Caterpillar 81 bulldozer hp; one Caterpillar Cat 824 wheeled dozer; four Caterpillar front-end loader; one Volvo front-end loader with 5.5–6 m^3 buckets; two Champion of 120 hp: one Caterpillar grader of 275 hp.

The proper transportation system has been seen that it would be selected among the shovel-truck (A_1), shovel-truck-in-pit crusher-belt conveyor (A_2), shovel-in-pit crusher-belt conveyor (A_3) and loader-truck (A_4) systems. The characteristic of the mine-site and the equipment technical futures are given in Table 1.

TABLE 1

Technical parameters calculated for each system

TABELA 1

Techniczne parametry wyznaczone dla każdego z systemów

Reserve	23.000.000 ton
Coal Production	1,200,000 ton/year (for power plant), 100.000 ton/year (for domestic uses)
Active workday	1 shift/day, 300 days/year, 12 h/day, 3600 h/year
Coal	Lignite, intermediate: clay
Coal density	1.5 ton/m^3
Average Coal thickness	19.3 m
Coal size	Max. 50 cm (run-of-mine), 10 cm (belt conveyor)
Coal Analyse	Moisture: % 26.6, Ash: % 26, Low Calorific Value: 2097 kcal/kg, Sulfide: % 2
Swell Factor (coal)	1,2 (conveying)
Blasting	Exist
Haulage distance	2.5 km. (A_1), 2 km belt conveying – 0.5 km truck haulage (A_2), 2.5 km (A_3), 2.5 km (A_4)
Average grade resistance	%3
Average rolling resistance	%2
Max. inverse grade	+%4
Dump level	Front Shovel: 7.5m. Truck (Loading Height: 3.78 m)
Bucket capacity	Hydraulic Excavator : 7.64 m^3
Bucket fill factor	%90
Operating weight	Front Shovel: 83 800 kg, Truck : 40 188 kg
Useful life	Front Shovel: 25 000 h. Loader: 20 000 h. Truck: 15 000 h. Conveyor: 24 000 h
Loading time	Hydraulic Excavator : 26 sec
Cycle time	17.85 min for 2.5 km (A_1), 7.2 min for 0.5 km (truck-conveyor)
Belt Conveyor	2 m/sec, 900 mm width, 2.5 km length (out of pit) 0.5 km (in-pit)
In-pit Crusher	350 ton/h
Capital cost	Truck : $400.000 Crusher: $700.000, Conveyor: 2.670.000 (2.5 km)
Operating cost	A_1 = $12/ton, A_2 = $6.80/ton, A_3 = $6.12/ton, A_4 = $11.72/ton

The following are some of the given linguistic results produced from various solution methods (linear programming, expert systems, etc.) and therefore presented by the experts to questions posed (what if…? or if..?, etc.) Each system has shown its own advantages. In this case, it did not appear that an easy solution to the problem could be obtained. From the solution point of view, application of the fuzzy set theory would be a proper choice, and therefore used in this paper.

— The road conditions differ from season to season. Thus the rolling resistance gives rather low point in dry season while it reaches the high in winter.
— Diggibility is not being difficult so the front-shovel can be selected unhesitatingly.
— The front-shovel as regards to the ground condition has more advantage (it is very wet and marshy especially in winter).
— Maximum material size is about 0.5 m. This shows the truck haulage to good advantage from the loading point of view.
— The front-shovel is a much better excavator in terms of the bench planned to have a 20-m height.
— All combinations (systems) are suitable in regard to the height of dump but the front-shovel can make much more safe loading.
— The haulage distance varies between 2000 and 2500 m. In this case A_2 can be considered as a better combination of loading-hauling system.
— A_2 is the better system in terms of the working stability.

The criteria of each operation is summarized in Table 2 and in the following, an optimum loading-hauling system selection procedure using Yager method is given.

3.1. The first Yager method [4]

Let $A = \{A_1, A_2, A_3, A_4\}$ be the set of alternative systems and $C = \{C_1, C_2, C_3, \ldots, C_m\}$ be the set of criteria. The decision-maker is then asked to define the membership grade of each criterion that is conferred with experts on this subject. Following that procedure the membership grade of each criterion is given in detail:

$C_1 = \{0.85/A_1, 0.90/A_2, 0.70/A_3, 0.65/A_4\}$ $C_{12} = \{0.75/A_1, 0.85/A_2, 0.90/A_3, 0.65/A_4\}$
$C_2 = \{1.00/A_1, 0.80/A_2, 0.80/A_3, 0.85/A_4\}$ $C_{13} = \{0.98/A_1, 0.90/A_2, 0.65/A_3, 0.95/A_4\}$
$C_3 = \{0.90/A_1, 0.88/A_2, 0.88/A_3, 0.90/A_4\}$ $C_{14} = \{0.60/A_1, 0.75/A_2, 0.80/A_3, 0.60/A_4\}$
$C_4 = \{0.60/A_1, 0.90/A_2, 0.95/A_3, 0.60/A_4\}$ $C_{15} = \{0.65/A_1, 0.80/A_2, 0.85/A_3, 0.65/A_4\}$
$C_5 = \{0.80/A_1, 0.90/A_2, 0.30/A_3, 0.70/A_4\}$ $C_{16} = \{1.00/A_1, 0.90/A_2, 0.65/A3, 1.00/A_4\}$
$C_6 = \{0.85/A_1, 0.90/A_2, 1.00/A_3, 0.85/A_4\}$ $C_{17} = \{0.70/A_1, 0.90/A_2, 1.00/A_3, 0.70/A_4\}$
$C_7 = \{0.50/A_1, 0.80/A_2, 0.85/A_3, 0.40/A_4\}$ $C_{18} = \{0.85/A_1, 0.75/A_2, 0.70/A_3, 0.85/A_4\}$
$C_8 = \{0.80/A_1, 0.95/A_2, 1.00/A_3, 0.80/A_4\}$ $C_{19} = \{0.70/A_1, 0.95/A_2, 1.00/A_3, 0.70/A_4\}$
$C_9 = \{0.78/A_1, 0.95/A_2, 1.00/A_3, 0.78/A_4\}$ $C_{20} = \{0.90/A_1, 0.80/A_2, 0.70/A_3, 0.95/A_4\}$
$C_{10} = \{0.75/A_1, 0.85/A_2, 0.90/A_3, 0.75/A_4\}$ $C_{21} = \{0.70/A_1, 0.85/A_2, 0.90/A_3, 0.70/A_4\}$
$C_{11} = \{0.80/A_1, 0.90/A_2, 0.30/A_3, 0.60/A_4\}$

Additionally, the mxm matrix (Fig. 2) was constructed to express the decision-makers' empirical estimate of importance for each criterion. Then, the maximum eigenvector was obtained using the Matlab (version 5.0). The judgment scale used here as: 1 Equally important; 1.5 weakly more important; 2 Strongly more important; 2.5 demonstrably more

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}	C_{21}
C_1	1	1/1,5	1,5	1/2	1/3	1/2	1/2	1/1,5	1/1,5	1	1/2,5	1	1/2,5	1/2,5	1/3	1/3	1/2	1/2	1/3	1/2,5	1/3
C_2	1,5	1	1	1/1,5	1/2.5	1/1,5	1/2	1	1/1,5	1/1,5	1/3	1/2	1/2,5	1/2,5	1/3	1/3	1/2	1/1,5	1/2,5	1/2,5	1/3
C_3	1/1,5	1	1	1/2	1/3	1	1/2	1/1,5	1/1,5	1,5	1/3	1/1,5	1/3	1/2,5	1/3	1/3	1/2	1/1,5	1/3	1/2,5	1/3
C_4	2	1,5	2	1	2	2	1,5	1,5	1,5	2,5	1/2	2,5	1	1	1/1,5	1/2	1	2	1/2	1/2	1/2,5
C_5	3	2,5	3	1/2	1	3	2	3	2	3	1	2	1,5	1,5	1	1	2,5	2,5	1	1	1/2,5
C_6	2	1,5	1	1/2	1/3	1	1/2	1	1/1,5	1	1/2,5	1/1,5	1/2	1/2	1/2,5	1/2,5	1	1	1/2	1/2	1/2,5
C_7	2	2	2	1/1,5	1/2	2	1	1,5	1	1,5	1/2	1	1/2	1/2	1/2,5	1/2,5	1	1	1/2	1/2	1/2,5
C_8	1,5	1	1,5	1/1,5	1/3	1	1/1,5	1	1/1,5	1	1/2,5	1/1,5	1/2,5	1/2,5	1/3	1/3	1	1	1/3	1/2,5	1/3
C_9	1,5	1,5	1,5	1/1,5	1/2	1,5	1	1,5	1	1,5	1/2	1	1/2	1/2	1/2,5	1/2,5	1	1	1/2	1/2	1/2,5
C_{10}	1	1,5	1/1,5	1/2,5	1/3	1	1/1,5	1	1/1,5	1	1/2,5	1	1/2	1/2	1/2,5	1/2,5	1	1	1/2,5	1/2	1/2,5
C_{11}	2,5	3	3	2	1	2,5	2	2,5	2	2,5	1	3	1	1	1/1,5	1/1,5	2,5	2,5	1	1	1/1,5
C_{12}	1	2	1,5	1/2,5	1/2	1,5	1	1,5	1	1	1/3	1	1/2	1/2	1/2,5	1/2,5	1	1	1/2	1/2	1/2,5
C_{13}	2,5	2,5	3	1	1/1,5	2	2	2,5	2	2	1	2	1	1	1/1,5	1/1,5	2,5	2,5	1	1	1/1,5
C_{14}	2,5	2,5	2,5	1	1/1,5	2	2	2,5	2	2	1	2	1	1	1/1,5	1/1,5	2	2	1/1,5	1	1/1,5
C_{15}	3	3	3	1,5	1	2,5	2,5	3	2,5	2,5	1,5	2,5	1,5	1,5	1	1	3	3	1	1,5	1
C_{16}	3	3	3	2	1	2,5	2,5	3	2,5	2,5	1,5	2,5	1,5	1,5	1	1	2,5	3	1	1	1/1,5
C_{17}	2	2	2	1	1/2,5	1	1	1	1	1	1/2,5	1	1/2,5	1/2	1/3	1/2,5	1	1	1/2	1/2	1/2,5
C_{18}	2	1,5	1,5	1/2	1/2,5	1	1	1	1	1	1/2,5	1	1/2,5	1/2	1/3	1/3	1	1	1/2	1/2	1/2,5
C_{19}	3	2,5	3	2	1	2	2	3	2	2,5	1	2	1	1,5	1	1	2	2	1	1	1/1,5
C_{20}	2,5	2,5	2,5	2	1	2	2	2,5	2	2	1	2	1	1	1/1,5	1	2	2	1	1	1/1,5
C_{21}	3	3	3	2,5	1,5	2,5	2,5	3	2,5	2,5	1,5	2,5	1,5	1,5	1	1,5	2,5	2,5	1,5	1,5	1

Fig. 2. Criterion comparisons

Rys. 2. Porównaniw kryteriów

important; 3 Absolutely more important. For the example described, Saaty's reciprocal matrices are follow in Figure 2.These matrices are constructed by expert team.

Hence, the maximum eigenvalue of the reciprocal matrix is $\lambda = 21.3642$. The weights of the criteria are finally obtained in the eigenvector of the matrix. Eigenvector = {0.0960, 0.0989, 0.0965, 0.2168, 0.2885, 0.1198, 0.1506, 0.1113, 0.1410, 0.1147, 0.2844, 0.1334, 0.2557, 0.2428, 0.3346, 0.3246, 0.1381, 0.1275, 0.2877, 0.2670, 0.3596} with $\lambda_{\max}$.

The eigenvector corresponds to the weights to be associated with the memberships of each attribute/feature/goal. Thus, the exponential weighting is $\alpha_1 = 0.0960$, $\alpha_2 = 0.0989$, $\alpha_3 = 0.0965$, $\alpha_4 = 0.2168$, $\alpha_5 = 0.2885$, $\alpha_6 = 0.1198$, $\alpha_7 = 0.1506$, $\alpha_8 = 0.1113$, $\alpha_9 = 0.1410$, $\alpha_{10} = 0.1147$, $\alpha_{11} = 0.2844$, $\alpha_{12} = 0.1334$, $\alpha_{13} = 0.2557$, $\alpha_{14} = 0.2428$, $\alpha_{15} = 0.3346$, $\alpha_{16} = 0.3246$, $\alpha_{17} = 0.1381$, $\alpha_{18} = 0.1275$, $\alpha_{19} = 0.2877$, $\alpha_{20} = 0.2670$, $\alpha_{21} = 0.3596$ and the final decision is obtained as follows:

$\mu_D(A) = \min(\mu_{C1}(A)^{\alpha 1}, \mu_{C2}(A)^{\alpha 2}, \ldots, \mu_{Cm}(A)^{\alpha m})$ for $\alpha > 0$ and the optimal decision, $\mu_D(A^*) = \max_A \mu_D(A)$ where A^* is the optimal decision.

C_1 = {0.85/A_1, 0.90/A_2, 0.70/A_3, 0.65/A_4}$^{0.0960}$ = {0.98/A_1,1.00/A_2,0.96/A_3,0.96/A_4}
C_2 = {1.00/A_1, 0.80/A_2, 0.80/A_3, 0.85/A_4}$^{0.0989}$ = {1.00/A_1,0.98/A_2,0.98/A_3,0.98/A_4}
C_3 = {0.90/A_1, 0.88/A_2, 0.88/A_3, 0.90/A_4}$^{0.0965}$ = {0.99/A_1,0.98/A_2,0.98/A_3,0.99/A_4}
C_4 = {0.60/A_1, 0.90/A_2, 0.95/A_3, 0.60/A_4}$^{0.2168}$ = {0.89/A_1,0.97/A_2,0.99/A_3,0.89/A_4}
C_5 = {0.80/A_1, 0.90/A_2, 0.30/A_3, 0.70/A_4}$^{0.2885}$ = {0.93/A_1,0.97/A_2,0.70/A_3,0.90/A_4}
C_6 = {0.85/A_1, 0.90/A_2, 1.00/A_3, 0.85/A_4}$^{0.1198}$ = {0.98/A_1,0.98/A_2,1.00/A_3,0.98/A_4}
C_7 = {0.50/A_1, 0.80/A_2, 0.85/A_3, 0.40/A_4}$^{0.1506}$ = {0.90/A_1,0.96/A_2,0.85/A_3,0.87/A_4}
C_8 = {0.80/A_1, 0.95/A_2, 1.00/A_3, 0.80/A_4}$^{0.1113}$ = {0.97/A_1,0.99/A_2,1.00/A_3,0.97/A_4}
C_9 = {0.78/A_1, 0.95/A_2, 1.00/A_3, 0.78/A_4}$^{0.1410}$ = {0.96/A_1,0.99/A_2,1.00/A_3,0.96/A_4}
C_{10} = {0.75/A_1, 0.85/A_2, 0.90/A_3,0.75/A_4}$^{0.1147}$ = {0.96/A_1,0.98/A_2,0.98/A_3,0.96/A_4}
C_{11} = {0.80/A_1, 0.90/A_2, 0.30/A_3,0.60/A_4}$^{0.2844}$ = {0.94/A_1,0.97/A_2,0.71/A_3,0.86/A_4}
C_{12} = {0.75/A_1, 0.85/A_2, 0.90/A_3,0.65/A_4}$^{0.1334}$ = {0.96/A_1,0.98/A_2,0.98/A_3,0.94/A_4}
C_{13} = {0.98/A_1, 0.90/A_2, 0.65/A_3,0.95/A_4}$^{0.2557}$ = {0.99/A_1,0.97/A_2,0.89/A_3,0.98/A_4}
C_{14} = {0.60/A_1, 0.75/A_2, 0.80/A_3,0.60/A_4}$^{0.2428}$ = {0.88/A_1,0.93/A_2,0.94/A_3,0.88/A_4}
C_{15} = {0.65/A_1, 0.80/A_2, 0.85/A_3,0.65/A_4}$^{0.3346}$ = {0.86/A_1,0.93/A_2,0.94/A_3,0.86/A_4}
C_{16} = {1.00/A_1, 0.90/A_2, 0.65/A_3,1.00/A_4}$^{0.3246}$ = {1.00/A_1,0.96/A_2,0.87/A_3,1.00/A_4}
C_{17} = {0.70/A_1, 0.90/A_2, 1.00/A_3,0.70/A_4}$^{0.1381}$ = {0.95/A_1,0.98/A_2,1.00/A_3,0.95/A_4}
C_{18} = {0.85/A_1, 0.75/A_2, 0.70/A_3,0.85/A_4}$^{0.1275}$ = {0.98/A_1,0.96/A_2,0.95/A_3,0.98/A_4}
C_{19} = {0.70/A_1, 0.95/A_2, 1.00/A_3,0.70/A_4}$^{0.2877}$ = {0.90/A_1,0.98/A_2,1.00/A_3,0.90/A_4}
C_{20} = {0.90/A_1, 0.80/A_2, 0.70/A_3,0.95/A_4}$^{0.2670}$ = {0.97/A_1,0.94/A_2,0.91/A_3,0.98/A_4}
C_{21} = {0.70/A_1, 0.85/A_2, 0.90/A_3,0.70/A_4}$^{0.3596}$ = {0.88/A_1,0.94/A_2,0.96/A_3,0.88/A_4}

$\mu_D(A)$ = {0.86/A_1, 0.93/A_2, 0.70/A_3, 0.86/A_4} and the optimal solution is,
$\mu_D(A^*)$ = 0.93/A_2.

The first Yager method (Yager 1978) follows the maxmin method of Bellman, Zadeh (Bascetin 2007) with the improvement of using Saaty's method of reciprocal matrix to express the criteria pair-wise comparison and the resulting eigenvector as the subjective weights for the criteria. The weighting procedure uses exponentials based on the definition of linguistic hedges proposed in (Bellman 1970; Zadeh 1973). The main drawback is still the problem that the minimum rule (intersection) could exclude one alternative just because one criteria's relative merit is quite small even if the other are very high (the and operator).

3.2. The second Yager method (Yager 1981)

Illustrating with the same example but considering that all ratings are done with fuzzy linguistic terms from the set R = {very low, low, average, high, very high}, the classifications are:

C_1 = {high/A_1, veryhigh/A_2, high/A_3, average/A_4}
C_2 = {veryhigh/A_1, high/A_2, high/A_3, veryhigh/A_4}
C_3 = {veryhigh/A$_1$, high/A_2, high/A_3, veryhigh/A_4}

C_4 = {average/A_1, veryhigh/A_2, veryhigh/A_3, average/A_4}
C_5 = {high/A_1, veryhigh/A_2, low/A_3, average/A_4}
C_6 = {high/A_1, veryhigh/A_2, veryhigh/A_3, high/A_4}
C_7 = {average/A_1, high/A_2, high/A_3, low/A_4}
C_8 = {high/A_1, veryhigh/A_2, veryhigh/A_3, high/A_4}
C_9 = {high/A_1, veryhigh/A_2, veryhigh/A_3, high/A_4}
C_{10} = {high/A_1, high/A_2, veryhigh/A_3, average/A_4}
C_{11} = {high/A_1, veryhigh/A_2, low/A_3, average/A_4}
C_{12} = {high/A_1, high/A_2, veryhigh/A_3, low/A_4}
C_{13} = {veryhigh/A_1, veryhigh/A_2, average/A_3, veryhigh/A_4}
C_{14} = {average/A_1, high/A_2, high/A_3, average/A_4}
C_{15} = {average/A_1, high/A_2, high/A_3, average/A_4}
C_{16} = {veryhigh/A_1, veryhigh/A_2, average/A_3, veryhigh/A_4}
C_{17} = {high/A_1, veryhigh/A_2, veryhigh/A_3, high/A_4}
C_{18} = {high/A_1, high/A_2, high/A_3, high/A_4}
C_{19} = {high/A_1, veryhigh/A_2, veryhigh/A_3, high/A_4}
C_{20} = {veryhigh/A_1, high/A_2, high/A_3, veryhigh/A_4}
C_{21} = {high/A_1, high/A_2, veryhigh/A_3, high/A_4}

and the preferences for the criteria are:

WC_1 = high	$\overline{WC}_1$ = low	WC_{12} = high	$\overline{WC}_{12}$ = low
WC_2 = high	$\overline{WC}_2$ = low	WC_{13}= high	$\overline{WC}_{13}$ = low
WC_3 = high	$\overline{WC}_3$ = low	WC_{14}= high	$\overline{WC}_{14}$ = low
WC_4 = high	$\overline{WC}_4$ = low	WC_{15} = high	$\overline{WC}_{15}$ = low
WC_5 = average	$\overline{WC}_5$ = average	WC_{16} = high	$\overline{WC}_{16}$ = low
WC_6 = veryhigh	$\overline{WC}_6$ = very low	WC_{17} = high	$\overline{WC}_{17}$ = low
WC_7 = high	$\overline{WC}_7$ = low	WC_{18} = high	$\overline{WC}_{18}$ = low
WC_8 = veyhigh	$\overline{WC}_8$ = verylow	WC_{19} = high	$\overline{WC}_{19}$ = low
WC_9 = veryhigh	$\overline{WC}_9$ = verylow	WC_{20} = high	$\overline{WC}_{20}$ = low
WC_{10} = high	$\overline{WC}_{10}$ = low	WC_{21} = high	$\overline{WC}_{21}$ = low
WC_{11} = high	$\overline{WC}_{11}$ = low		

Thus, using the operator $\vee$ for max,
C_1 = low $\vee$ {high/A_1, veryhigh/A_2, high/A_3, average/A_4}
C_2 = low $\vee$ {veryhigh/A_1, high/A_2, high/A_3, veryhigh/A_4}
C_3 = low $\vee$ {veryhigh/A_1, high/A_2, high/A_3, veryhigh/A_4}
C_4 = low $\vee$ {average/A_1, veryhigh/A_2, veryhigh/A_3, average/A_4}
C_5 = average $\vee$ {high/A_1, veryhigh/A_2, low/A_3, average/A_4}
C_6 = verylow $\vee$ {high/A_1, veryhigh/A_2, veryhigh/A_3, high/A_4}
C_7 = low $\vee$ {average/A_1, high/A_2, high/A_3, low/A_4}
C_8 = verylow $\vee$ {high/A_1, veryhigh/A_2, veryhigh/A_3, high/A_4}
C_9 = verylow $\vee$ {high/A_1, veryhigh/A_2, veryhigh/A_3, high/A_4}

C_{10} = low $\vee$ {high/A_1, high/A_2, veryhigh/A_3, average/A_4}
C_{11} = low $\vee$ {high/A_1, veryhigh/A_2, low/A_3, average/A_4}
C_{12} = low $\vee$ {high/A_1, high/A_2, veryhigh/A_3, low/A_4}
C_{13} = low $\vee$ {veryhigh/A_1, veryhigh/A_2, average/A_3, veryhigh/A_4}
C_{14} = low $\vee$ {average/A_1, high/A_2, high/A_3, average/A_4}
C_{15} = low $\vee$ {average/A_1, high/A_2, high/A_3, average/A_4}
C_{16} = low $\vee$ {veryhigh/A_1, veryhigh/A_2, average/A_3, veryhigh/A_4}
C_{17} = low $\vee$ {high/A_1, veryhigh/A_2, veryhigh/A_3, high/A_4}
C_{18} = low $\vee$ {high/A_1, high/A_2, high/A_3, high/A_4}
C_{19} = low $\vee$ {high/A_1, veryhigh/A_2, veryhigh/A_3, high/A_4}
C_{20} = low $\vee$ {veryhigh/A_1, high/A_2, high/A_3, veryhigh/A_4}
C_{21} = low $\vee$ {high/A_1, high/A_2, veryhigh/A_3, high/A_4}

and the decision function and the optimal final decision are:

D = {average/A_1, high/A_2, low/A_3, low/A_4}

$D^* = \min(D)$ › A_2

In summary the second Yager method result is similar with the other proposed by the same author. The main advantage of this method is that avoids the elimination of alternatives because the weights are very small, since it uses the max operator to determine the best alternative classification. The importance of the alternative methods proposed by Yager, lies in the fact that depending on the information available or the decision maker style any method could be used.

3.3. The analytic hierarchy process

The same example was applied to the analytic hierarchy process. Let $A = \{A_1, A_2, A_3, A_4\}$ be the set of alternative systems and $C = \{C_1, C_2, C_3, \ldots, C_m\}$ be the set of Criterion. The judgement scale used let is given as: 1. Equally important; 1.5 weakly more important; 2. Strongly more important; 2.5 demonstrably more important, 3. Absolutely more important. For the example described, Saaty's reciprocal matrices are follow in Figure 2. The other comparisons for alternatives according to each criterion are follow in below. These matrices are constructed by expert team.

C_1 Criterion

	A_1	A_2	A_3	A_4
A_1	1	1/1,5	2,5	2
A_2	1,5	1	3	2
A_3	1/2,5	1/3	1	1/2
A_4	1/2	1/2	2	1

C_2 Criterion

	A_1	A_2	A_3	A_4
A_1	1	2,5	2,5	1/2
A_2	1/2,5	1	1	1/3
A_3	1/2,5	1	1	1/3
A_4	2	3	3	1

C_3 Criterion

	A_1	A_2	A_3	A_4
A_1	1	2	2	1/2
A_2	1/2	1	1	1/2,5
A_3	1/2	1	1	1/2,5
A_4	2	2,5	2,5	1

C_4 Criterion

	A_1	A_2	A_3	A_4
A_1	1	1/2	1/2,5	1
A_2	2	1	1/2	2
A_3	2,5	2	1	2,5
A_4	1	1/2	1/2,5	1

C_5 Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/1,5 & 1/2,5 & 1 \\ A_2 & 1,5 & 1 & 3 & 1,5 \\ A_3 & 2,5 & 1/3 & 1 & 2,5 \\ A_4 & 1 & 1/1,5 & 1/2,5 & 1 \end{array}$$

C_6 Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/1,5 & 1/2 & 1 \\ A_2 & 1,5 & 1 & 1/2 & 1,5 \\ A_3 & 2 & 2 & 1 & 2 \\ A_4 & 1 & 1/1,5 & 1/2 & 1 \end{array}$$

C_7 Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/2 & 1/2,5 & 1,5 \\ A_2 & 2 & 1 & 1/1,5 & 2 \\ A_3 & 2,5 & 1,5 & 1 & 2,5 \\ A_4 & 1/1,5 & 1/2 & 1/2,5 & 1 \end{array}$$

C_8 Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/2 & 1/2,5 & 1 \\ A_2 & 2 & 1 & 1/1,5 & 2 \\ A_3 & 2,5 & 1,5 & 1 & 2,5 \\ A_4 & 1 & 1/2 & 1/2,5 & 1 \end{array}$$

C_9 Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/2 & 1/3 & 1 \\ A_2 & 2 & 1 & 1,5 & 2 \\ A_3 & 3 & 1/1,5 & 1 & 3 \\ A_4 & 1 & 1/2 & 1/3 & 1 \end{array}$$

C_{10} Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/1,5 & 1/2,5 & 1,5 \\ A_2 & 1,5 & 1 & 1/1,5 & 1,5 \\ A_3 & 2,5 & 1,5 & 1 & 2,5 \\ A_4 & 1/1,5 & 1/1,5 & 1/2,5 & 1 \end{array}$$

C_{11} Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/1,5 & 2 & 1,5 \\ A_2 & 1,5 & 1 & 2,5 & 1,5 \\ A_3 & 1/2 & 1/2,5 & 1 & 1/2 \\ A_4 & 1/1,5 & 1/1,5 & 2 & 1 \end{array}$$

C_{12} Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/1,5 & 1/2,5 & 2 \\ A_2 & 1,5 & 1 & 1/1,5 & 1,5 \\ A_3 & 2,5 & 1,5 & 1 & 2,5 \\ A_4 & 1/2 & 1/1,5 & 1/2,5 & 1 \end{array}$$

C_{13} Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1 & 3 & 1 \\ A_2 & 1 & 1 & 2,5 & 1/1,5 \\ A_3 & 1/3 & 1/2,5 & 1 & 1/3 \\ A_4 & 1 & 1,5 & 3 & 1 \end{array}$$

C_{14}Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/2 & 1/2,5 & 1 \\ A_2 & 2 & 1 & 1/1,5 & 2 \\ A_3 & 2,5 & 1,5 & 1 & 2,5 \\ A_4 & 1 & 1/2 & 1/2,5 & 1 \end{array}$$

C_{15} Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/2,5 & 1/3 & 1 \\ A_2 & 2,5 & 1 & 1/1,5 & 2,5 \\ A_3 & 3 & 1,5 & 1 & 3 \\ A_4 & 1 & 1/2,5 & 1/3 & 1 \end{array}$$

C_{16} Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 2 & 3 & 1/1,5 \\ A_2 & 1/2 & 1 & 2,5 & 1/2,5 \\ A_3 & 1/3 & 1/2,5 & 1 & 1/3 \\ A_4 & 1,5 & 2,5 & 3 & 1 \end{array}$$

C_{17} Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/2 & 1/3 & 1 \\ A_2 & 2 & 1 & 1/1,5 & 2 \\ A_3 & 3 & 1,5 & 1 & 3 \\ A_4 & 1 & 1/2 & 1/3 & 1 \end{array}$$

C_{18} Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1,5 & 2 & 1 \\ A_2 & 1/1,5 & 1 & 1,5 & 1/1,5 \\ A_3 & 1/2 & 1/1,5 & 1 & 1/2 \\ A_4 & 1 & 1,5 & 2 & 1 \end{array}$$

C_{19} Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/2,5 & 1/3 & 1 \\ A_2 & 2,5 & 1 & 1/1,5 & 2,5 \\ A_3 & 3 & 1,5 & 1 & 3 \\ A_4 & 1 & 1/2,5 & 1/3 & 1 \end{array}$$

C_{20} Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 2 & 2,5 & 1/1,5 \\ A_2 & 1/2 & 1 & 2 & 1/2,5 \\ A_3 & 1/2,5 & 1/2 & 1 & 1/3 \\ A_4 & 1,5 & 2,5 & 3 & 1 \end{array}$$

C_{21} Criterion

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 1 & 1/2 & 1/2,5 & 1 \\ A_2 & 2 & 1 & 1/1,5 & 2 \\ A_3 & 2,5 & 1,5 & 1 & 2,5 \\ A_4 & 1 & 1/2 & 1/2,5 & 1 \end{array}$$

To solve the reciprocal matrices it was used the maximum eigenvalue and eigenvectors. The eigenvector corresponding to the maximum eigenvalue is a cardinal ratio scale for the elements compared. The eigenvectors are then normalised to ensure consistency. After obtaining the normalised eigenvectors for each matrix, the vectors of the upper level became the members of the full matrix of weights of alternatives for each Criterion. This last matrix of vectors is then multiplied by the matrix of weights of the Criterion comparison (the eigenvector of the Criterion comparison). The intermediate results are in below.

TABLE 2

Criteria of each operation

TABELA 2

Kryteria każdej operacji

Criterion	Operation	Criterion	Operation
C1	Production	C12	Useful Life
C2	Material Size	C13	Flexibility
C3	Moisture	C14	Availability
C4	Haulage Distance	C15	Utilization
C5	The Ground Condition	C16	Mobility
C6	Haul Road Condition	C17	Continuous
C7	Environment (dust, noisy, etc)	C18	Support
C8	Grade	C19	Net to Tare Ratio
C9	Average Rolling Resistance	C20	Capital Cost
C10	Weather Conditions	C21	Operating Cost
C11	Working Stability		

Cretiron Comparisons

$$\begin{bmatrix}0{,}0960\\0{,}0989\\0{,}0965\\0{,}2168\\0{,}2885\\0{,}1198\\0{,}1506\\0{,}1113\\0{,}1410\\0{,}1147\\0{,}2844\\0{,}1334\\0{,}2557\\0{,}2428\\0{,}3346\\0{,}3246\\0{,}1381\\0{,}1275\\0{,}2877\\0{,}2670\\0{,}3596\end{bmatrix} \quad \textit{Normalized} \quad \begin{bmatrix}0{,}0230\\0{,}0240\\0{,}0230\\0{,}0520\\0{,}1000\\0{,}0300\\0{,}0400\\0{,}0300\\0{,}0300\\0{,}0270\\0{,}0700\\0{,}0300\\0{,}0600\\0{,}0500\\0{,}0800\\0{,}0700\\0{,}0330\\0{,}0300\\0{,}0700\\0{,}0600\\0{,}0800\end{bmatrix}$$

C_1 Criterion

$$\begin{bmatrix}0{,}5629\\0{,}7182\\0{,}2110\\0{,}3504\end{bmatrix} \quad \textit{normalized} \quad \begin{bmatrix}0{,}305\\0{,}400\\0{,}114\\0{,}200\end{bmatrix}$$

C_2 Criterion

$$\begin{bmatrix}0{,}5628\\0{,}2377\\0{,}2377\\0{,}7552\end{bmatrix} \quad \textit{normalized} \quad \begin{bmatrix}0{,}314\\0{,}130\\0{,}130\\0{,}421\end{bmatrix}$$

C_3 Criterion

$$\begin{bmatrix}0{,}5406\\0{,}2826\\0{,}2826\\0{,}7403\end{bmatrix} \quad \textit{normalized} \quad \begin{bmatrix}0{,}293\\0{,}153\\0{,}153\\0{,}400\end{bmatrix}$$

C_4 Criterion

$$\begin{bmatrix}0{,}2750\\0{,}4919\\0{,}7790\\0{,}2750\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}150\\0{,}270\\0{,}430\\0{,}150\end{bmatrix}$$

C_5 Criterion

$$\begin{bmatrix}0{,}3306\\0{,}7365\\0{,}5228\\0{,}2739\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}177\\0{,}395\\0{,}280\\0{,}147\end{bmatrix}$$

C_6 Criterion

$$\begin{bmatrix}0{,}3369\\0{,}4588\\0{,}7500\\0{,}3369\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}200\\0{,}243\\0{,}400\\0{,}200\end{bmatrix}$$

C_7 Critetria

$$\begin{bmatrix}0{,}2826\\0{,}5406\\0{,}7403\\0{,}2826\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}153\\0{,}300\\0{,}400\\0{,}153\end{bmatrix}$$

C_8 Criterion

$$\begin{bmatrix}0{,}2826\\0{,}5406\\0{,}7403\\0{,}2826\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}153\\0{,}300\\0{,}400\\0{,}153\end{bmatrix}$$

C_9 Criterion

$$\begin{bmatrix}0{,}2628\\0{,}6590\\0{,}6539\\0{,}2628\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}143\\0{,}360\\0{,}355\\0{,}143\end{bmatrix}$$

C_{10} Criterion

$$\begin{bmatrix}0{,}3115\\0{,}4799\\0{,}7587\\0{,}3115\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}167\\0{,}257\\0{,}400\\0{,}167\end{bmatrix}$$

C_{11} Criterion

$$\begin{bmatrix}0{,}6551\\0{,}6153\\0{,}228\\0{,}3776\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}350\\0{,}330\\0{,}120\\0{,}200\end{bmatrix}$$

C_{12} Criterion

$$\begin{bmatrix}0{,}3733\\0{,}4753\\0{,}7521\\0{,}2627\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}200\\0{,}255\\0{,}403\\0{,}141\end{bmatrix}$$

C_{13} Criterion

$$\begin{bmatrix}0{,}5684\\0{,}4915\\0{,}1976\\0{,}6296\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}300\\0{,}260\\0{,}104\\0{,}333\end{bmatrix}$$

C_{14} Criterion

$$\begin{bmatrix}0{,}2826\\0{,}5406\\0{,}7403\\0{,}2826\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}153\\0{,}300\\0{,}400\\0{,}153\end{bmatrix}$$

C_{15} Criterion

$$\begin{bmatrix}0{,}2377\\0{,}5628\\0{,}7552\\0{,}2377\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}132\\0{,}314\\0{,}420\\0{,}132\end{bmatrix}$$

C_{16} Criterion

$$\begin{bmatrix}0{,}6949\\0{,}3407\\0{,}1847\\0{,}6058\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}380\\0{,}186\\0{,}101\\0{,}331\end{bmatrix}$$

C_{17} Criterion

$$\begin{bmatrix}0{,}2582\\0{,}5164\\0{,}7746\\0{,}2582\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}143\\0{,}300\\0{,}430\\0{,}143\end{bmatrix}$$

C_{18} Criterion

$$\begin{bmatrix}0{,}6077\\0{,}4174\\0{,}2952\\0{,}6077\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}315\\0{,}216\\0{,}153\\0{,}315\end{bmatrix}$$

C_{19} Criterion

$$\begin{bmatrix}0{,}2377\\0{,}5628\\0{,}7552\\0{,}2377\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}132\\0{,}314\\0{,}420\\0{,}132\end{bmatrix}$$

C_{20} Criterion

$$\begin{bmatrix}0{,}5477\\0{,}3246\\0{,}2072\\0{,}7428\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}300\\0{,}200\\0{,}113\\0{,}400\end{bmatrix}$$

C_{21} Criterion

$$\begin{bmatrix}0{,}2826\\0{,}5406\\0{,}7403\\0{,}2826\end{bmatrix} \xrightarrow{normalized} \begin{bmatrix}0{,}153\\0{,}300\\0{,}400\\0{,}153\end{bmatrix}$$

Criterion Comparisons For Each Alternative (Matrice A)

$$\begin{bmatrix}
0{,}305 & 0{,}314 & 0{,}293 & 0{,}150 & 0{,}177 & 0{,}200 & 0{,}153 & 0{,}153 & 0{,}143 & 0{,}167 & 0{,}350 & 0{,}200 & 0{,}300 & 0{,}153 & 0{,}132 & 0{,}380 & 0{,}143 & 0{,}315 & 0{,}132 & 0{,}300 & 0{,}153\\
0{,}400 & 0{,}130 & 0{,}153 & 0{,}270 & 0{,}395 & 0{,}243 & 0{,}300 & 0{,}300 & 0{,}360 & 0{,}257 & 0{,}330 & 0{,}255 & 0{,}260 & 0{,}300 & 0{,}314 & 0{,}186 & 0{,}300 & 0{,}216 & 0{,}314 & 0{,}200 & 0{,}300\\
0{,}114 & 0{,}130 & 0{,}153 & 0{,}430 & 0{,}280 & 0{,}400 & 0{,}400 & 0{,}400 & 0{,}355 & 0{,}400 & 0{,}120 & 0{,}403 & 0{,}104 & 0{,}400 & 0{,}420 & 0{,}101 & 0{,}430 & 0{,}153 & 0{,}420 & 0{,}113 & 0{,}400\\
0{,}200 & 0{,}421 & 0{,}400 & 0{,}150 & 0{,}147 & 0{,}200 & 0{,}153 & 0{,}153 & 0{,}143 & 0{,}167 & 0{,}200 & 0{,}141 & 0{,}333 & 0{,}153 & 0{,}132 & 0{,}331 & 0{,}143 & 0{,}315 & 0{,}132 & 0{,}400 & 0{,}153
\end{bmatrix}$$

Criterion Weights (Matrice B)

$$\begin{bmatrix} 0{,}0230 \\ 0{,}0240 \\ 0{,}0230 \\ 0{,}0520 \\ 0{,}1000 \\ 0{,}0300 \\ 0{,}0400 \\ 0{,}0300 \\ 0{,}0300 \\ 0{,}0270 \\ 0{,}0700 \\ 0{,}0300 \\ 0{,}0600 \\ 0{,}0500 \\ 0{,}0800 \\ 0{,}0700 \\ 0{,}0330 \\ 0{,}0300 \\ 0{,}0700 \\ 0{,}0600 \\ 0{,}0800 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 0{,}1990 \\ 0{,}2915 \\ 0{,}3326 \\ 0{,}2306 \end{bmatrix} \qquad A_2 = 0{,}2915$$

$A_3 = 0{,}3326$

$A_4 = 0{,}2306$

The best alternative is thus A_3 (shovel-in-pit crusher-belt conveyor).

4. Conclusions

Equipment selection is one of the most important factor in open pit design and production planning. Equipment selection also affects economic considerations in open-pit design, specifically overburden, waste rock and ore mining costs and cost escalation parameters as a function of plan location and depth. Furthermore, the equipment selection is a complex multi-person, multi-criteria decision problem. The group decision-making process can be improved by a systematic and logical approach to assess priorities based on the inputs of several people from different functional areas within the mine company. This paper has discussed decision making in a fuzzy environment (uncertain data-linguistic variables involved in the systems) for solving multiple attribute problems of optimum transportation systems in final design. The most important approaches and basic concepts were introduced. This paper offers a new elicitation method for assigning weights. The proposed fuzzy aiding tool gives the decision-maker the flexibility of selecting the importance process that best suits him/her, and in case of incomplete knowledge it offers the possibility of an automatic algorithm based on past cases.

In addition to this, the analytic hierarchy process that is studied in this paper is quite consistent, structured and intuitive. The main problem lies in the fact that since all comparisons are done by importance comparisons, it loses the possible nuances of describing criteria as fuzzy sets. Another drawback is that the exhaustive pair-wise comparison, required to ensure consistency, is time consuming if there are many criteria. So the selected best alternative (A_3) is different from fuzzy set theory (the best alternative A_2). However, the analytical hierarchy process really represents an alternative to fuzzy set approaches, for obtaining alternative ratings in multiple attribute problems dealing with uncertainty.

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THE STUDY OF DECISION MAKING TOOLS FOR EQUIPMENT SELECTION IN MINING ENGINEERING OPERATIONS

Key words

Open pit mining, Equipment Selection, Fuzzy Logic, Analytical Hierarchy Process

Abstract

Equipment selection decisions which are the most important stage of short and long term surface mine planning have radically influenced the economic life of any mining scenario. Furthermore, equipment selection is also an complex multi-person, multi-criteria decision problem. This study has been directed to the research of an optimal loading-hauling system to a power station to be established in an open pit coal mine located Orhaneli, west of Turkey. Within this paper, the Analytical Hierarchy Process(AHP, non-fuzzy method) and different fuzzy methods are presented as an innovative tool for criteria aggregation in mining decision problems.

The paper is divided into four main sections. The first section provides an overview of the underlying concepts and theories of AHP and multiple attribute decision making in a fuzzy environment and the scope of this type of search. The second section introduces few applications of fuzzy set theory and AHP to mining industry problems reported in the literature. Some of these applications are briefly reviewed in the paper. The third section presents a case study which illustrate the application of the system for equipment selection in surface mining. Details of alternative systems and their criterion of each operation are also given. Finally, the fourth section presents the concluding remarks.

ANALIZA NARZĘDZI PODEJMOWANIA DECYZJI DO WYBORU WYPOSAŻENIA W ZAKŁADACH GÓRNICZYCH

Słowa kluczowe

Górnictwo odkrywkowe, wybór wyposażenia, teoria zbiorów rozmytych, metoda AHP

Streszczenie

Decyzje dotyczące wyboru wyposażenia, które są najistotniejszym elementem zarówno krótkoterminowego jak i długoterminowego planowania w kopalni odkrywkowej, mają wpływ na ekonomikę kopalni.

Co więcej, wybór wyposażenia jest trudnym problemem decyzyjnym, zależnym od wielu kryteriów i wielu osób. Ten artykuł opisuje badania dotyczące optymalnego systemu załadowczo-transportowego do elektrowni, jaki ma być wybudowany w kopalni odkrywkowej w Orhaneli w zachodniej części Turcji. Przedstawiono metodę AHP oraz różne metody wykorzystujące teorię zbiorów rozmytych jako innowacyjne narzędzie do agregacji kryteriów wyboru w problemach podejmowania decyzji w górnictwie.

Artykuł jest podzielony na cztery części. Część pierwsza to przegląd koncepcji i teorii AHP oraz metod wielokryterialnych podejmowania decyzji w warunkach rozmytych oraz zakres tego typu badań. Druga część przytacza pokrótce znalezione w literaturze zastosowania teorii zbiorów rozmytych i AHP do rozwiązywania problemów górnictwa . W trzeciej części przedstawiono przykład, który ilustruje zastosowanie tych narzędzi do wyboru wyposażenia w górnictwie odkrywkowym. Przedstawiono szczegóły alternatywnych rozwiązań i kryteria dla każdej z operacji. W końcu, czwarta część zawiera uwagi podsumowujące.