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## The case of two-component revenue while determining boundary values in the Costs-Volume-Profit (CVP) Analysis

### Introduction

The concept of boundary values is related to the CVP (Costs-Volume-Profit) analysis which is based on the profit equation (1). This formula takes quantity of output into account adds useful information for examining the effects of sale revenue, costs and volume on operating profits (Deakin, Maher 1987).

$$P_o = R_S - C_o \quad (1)$$

where:

- $P_o$  – operating profit,
- $R_S$  – sale revenues,
- $C_o$  – operating costs.

Cost-volume-profit (CVP) analysis is applied mainly to short-run situations (decisions). It is also sometimes referred to as break-even analysis, but this term is somewhat narrow in scope. CVP analysis covers a broader range of situations (Wilkinson 2005).

Therefore, CVP analysis involves evaluation of interconnections among six economic parameters (Czopek 2003):

- operating profit,

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- quantity of output,
- sale volume,
- sale price,
- fixed costs,
- variable costs.

Boundary values of parameters listed above mean their amount/value, for which operating profit is zero, assuming constancy of the other ones. The so-called break-even point, quantitative –  $BEP$  and value-related –  $BEP'$  is a generally known boundary values. Operating costs  $C_o$  divided into fixed and variable costs are starting point when determining these values:

$$C_o = C_f + c_v \cdot X \quad (2)$$

where:

- $C_o$  – operating costs,
- $C_f$  – fixed costs, remain unaltered with changing exploitation scale,
- $c_v$  – variable unit costs,
- $X$  – sale volume.

The CVP method assumes that when determining sale revenue, unit price for sale is constant regardless of sale volume. Then, the value of sale revenue is given by the following formula:

$$R_S = p \cdot X \quad (3)$$

where:

- $R_S$  – sale revenue,
- $p$  – unit sale price,
- $X$  – sale volume.

When we compare operating profit  $P_o$  to zero, where  $P_o$  is the difference between sale revenues- $R_S$  and operating costs- $C_o$ , it will be possible to determine all boundary values. Then, boundary sale volume – quantitative break-even point, will be determined by the following formula:

$$BEP = \frac{C_f}{p - c_v} \quad (4)$$

and value-related break-even point will be represented by the equation:

$$BEP' = p \cdot BEP \quad (5)$$

Using equations (1), (2), (3) and (4) we are able to determine formulas for the other boundary values, that is: boundary price, boundary fixed costs and boundary variable unit costs.

## 2. Two-component revenue from sale

Break-even point determined using formula (4) is correct in decisive majority of cases in economic turnover, however not in all situations. For example, it is different in companies distributing gas and power, or those providing telecommunication services. Conventionally, **revenue** of these companies consists of two parts; fixed – independent of sale or service scale, and variable – changing with sale level. This is the effect of tariff system being used by these companies, which consists of a set of prices and charge rates, and conditions for applying them to their respective consumers.

Good example of this is the tariff system used in high-methane natural gas turnover, applied to consumers in various tariff/charge groups. If we take a most representative gas buyer, an individual consumer and a micro businessman, then total revenue of gas distributing company will consist of four parts per 1 month:

- revenue from sold gas,
- revenue from variable network charge,
- standing charge,
- revenue from fixed network charge.

This revenue may be expressed using the following formula:

$$R_S = p \cdot X + p_v \cdot X + s_t + p_f \quad (6)$$

where:

- $S$  – gas distributor's sale revenue for single consumer in a given tariff group [PLN],
- $p$  – price of gas [PLN/m<sup>3</sup>],
- $X$  – volume of sold gas [m<sup>3</sup>],
- $p_v$  – variable network charge [PLN/m<sup>3</sup>],
- $s_t$  – standing charge [PLN/month],
- $p_f$  – fixed network charge [PLN/month].

After having entered additional symbols:

$$V = (p + p_v) \quad (7)$$

where:

- $V$  – sum of variable revenue parameters

$$F = (s_t + p_f) \quad (8)$$

where:

$F$  – sum of fixed revenue parameters,

the **revenue** given by formula (6) will be as follows:

$$R_S = V \cdot X + F \quad (9)$$

In case of two-component revenue, average sale price  $\bar{p}$  being a division of revenue given in formula (9) by sale volume  $X$ , may be then expressed as follows:

$$\bar{p} = V + \frac{F}{X} \quad (10)$$

This will not be a fixed price, as in case of equation (3), because it consists of fixed part- $V$  and variable part –  $F/X$ , therefore it will be changing hyperbolically with increasing sale volume (Fig. 1).

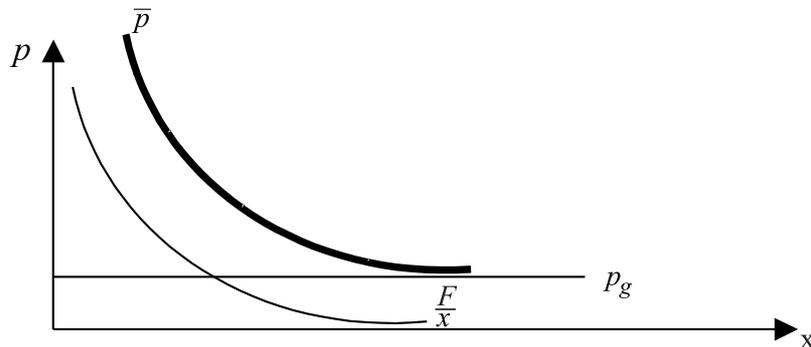


Fig. 1. Change of average price value for two-component revenue

Rys. 1. Zmiana średniej wartości ceny przy dwuskładnikowym przychodzie

Therefore, in this case it is not possible to use average price  $\bar{p}$ , when determining break-even point according to formula (4).

When the formula (10) is analysed, it is occurred that in case that the consumer does not buy any gas in analysed time period, then he is obligated to pay the fixed charges as:  $s_t$  – standing charge, and  $p_f$  – fixed network charge. In that case it is difficult to say about average sale price, because the gas wasn't sold (company) or buy (consumer). Taking into account this situation, this formula is applicable in most cases when consumer actually consumes (buys) the gas.

### 3. Boundary values for two-component revenue

According to primary rule of the CVP analysis, the boundary value is parameter value, for which operating profit is zero (1). Using formulas (2) and (9), we may express the above condition as follows (Trzaskuś-Żak 2005):

$$P_0 = [(V \cdot X + F) - (C_f + c_v \cdot X)] = 0 \quad (11)$$

Condition (11) is met if:

$$V \cdot X + F = C_f + c_v \cdot X \quad (12)$$

Therefore, expression (12) provides the basis for determining all boundary values for two-component revenue. In case of boundary value of sale  $X$ , that is quantitative break-even point  $BEP$ , transformation of formula (12) shows that break-even point is defined by the following equation:

$$BEP = \frac{C_f - F}{V - c_v} \quad (13)$$

Compared to conventional method used to determine break-even point, two distinct changes occur in formula (13) in gas distribution conditions:

- a new parameter has occurred in numerator, fixed revenue  $-F$ , as a result of this we deal with difference between fixed costs and fixed revenue,
- variable price component  $-V$  appears in denominator instead of average sale price  $p$ .

**Value-related break-even point**  $- BEP'$ , is a product of quantitative break-even point and average sale price, that is:

$$BEP' = BEP \cdot \bar{p} \quad (14)$$

and after transformation:

$$BEP' = \frac{C_f - F}{V - c_v} \cdot \left( V + \frac{F}{X} \right) \quad (15)$$

Fig. 2 illustrates graphical interpretation of quantitative and value-related break-even point taking into account two-component revenue.

Using formula 12, it is possible to determine other boundary values, that is (Trzaskuś-Żak 2005):

- **boundary value of fixed revenue**  $- F_b$

$$F_b = C_f - X \cdot (V - c_v) \quad (16)$$

— **boundary value of fixed costs** –  $C_{fb}$

$$C_{fb} = F + X \cdot (V - c_v) \quad (17)$$

— **boundary value of variable price component** –  $V_b$

$$V_b = \frac{C_o - F}{X} \quad (18)$$

— **boundary variable unit costs** –  $c_{vb}$

$$c_{vb} = \frac{F - C_f + X \cdot V}{X} \quad (19)$$

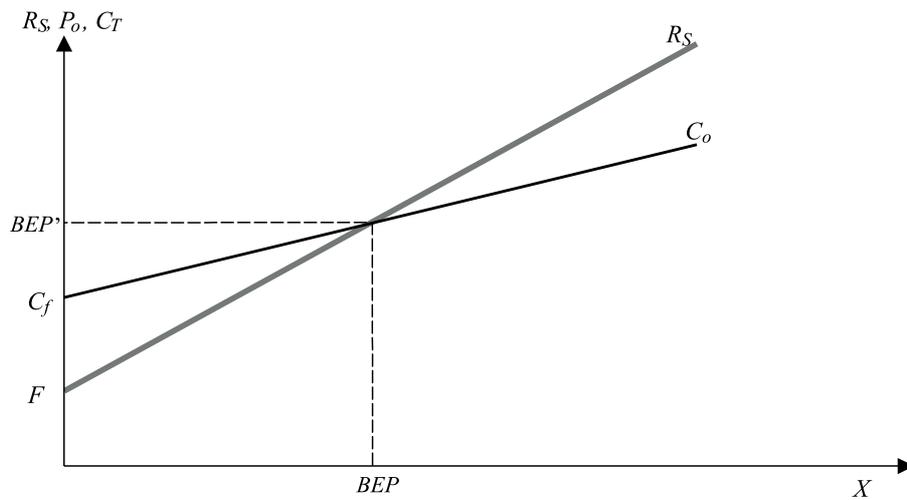


Fig. 2. Quantitative and value-related break-even point taking into account two components of revenue (Trzaskuś-Żak 2005)

$C_f$  – fixed costs [PLN];  $F$  – fixed revenue [PLN];  $R_S$  – sale revenue [PLN];  $C_o$  – operating costs [PLN];  
 $X$  – gas sale volume (output) [m<sup>3</sup>];  $BEP$  – quantitative break-even point [m<sup>3</sup>];  
 $BEP'$  – value-related break-even point [PLN]

Rys. 2. Ilościowy i wartościowy próg rentowności przy uwzględnieniu dwóch składników przychodu (Trzaskuś-Żak 2005)

$C_f$  – koszty stałe [PLN];  $F$  – przychód stały [PLN];  $R_S$  – przychody ze sprzedaży [PLN];  
 $C_o$  – koszty operacyjne [PLN];  $X$  – wielkość sprzedaży gazu [m<sup>3</sup>];  $BEP$  – ilościowy próg rentowności [m<sup>3</sup>];  
 $BEP'$  – wartościowy próg rentowności [PLN]

### 3. Comparison of boundary values obtained using both methods

The system used to settle distribution prices for gas indicates that using it to determine boundary values in conventional form is imprecise. First of all because according to formula (3) sale revenue has a limitation in the CVP analysis:

$$R_S \geq 0 \quad (20)$$

whereas, in the tariff system, sale revenue is a sum of fixed and variable **revenue**, therefore determinancy domain is:

$$R_S \geq F \quad (21)$$

The above-mentioned inaccuracy of computations results from the fact that when determining the basic boundary value, that is break-even-point, companies distributing gas, power, or providing telecommunication services use conventional formula (4), while the value of price occurring in denominator is determined according to the formula (10). Therefore, break-even point (conventionally marked  $BEP_1$ ) is then defined by the following equation:

$$BEP_1 = \frac{C_f}{\bar{p} - c_v} \quad (22)$$

Therefore, comparison of boundary values determined using formulas (13) and (22) involves verifying whether:

— these values are equal regardless of their determination method, that is:

$$BEP = BEP_1 \quad (23)$$

— the value determined using formula (22) is lower than that determined using (13), that is:

$$BEP > BEP_1 \quad (24)$$

— the value determined using formula (22) is higher than that determined using (13), that is:

$$BEP_1 > BEP \quad (25)$$

This may be represented graphically, as in Fig. 3.

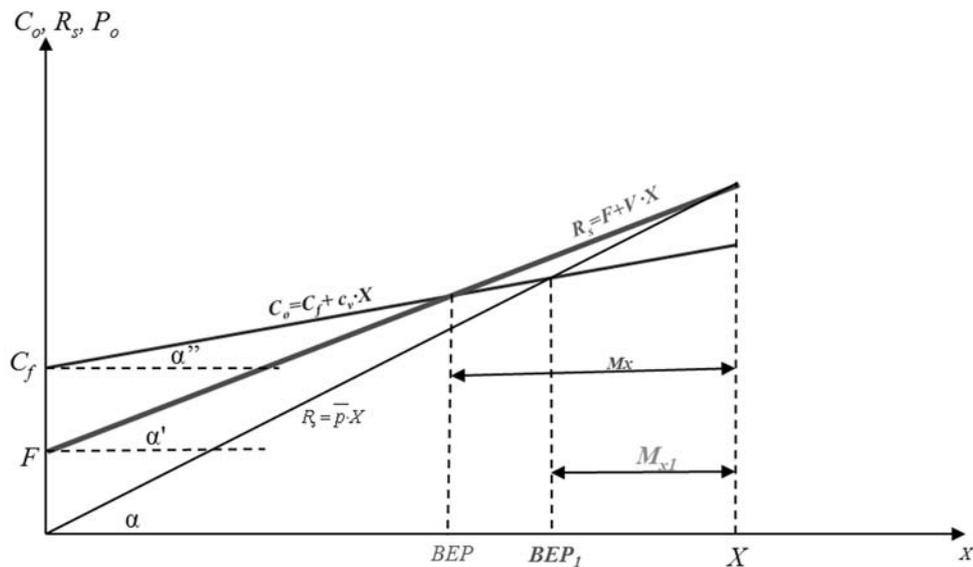


Fig. 3. Graphical comparison of break-even point determined using two methods

Rys. 3. Graficzne porównanie prognozy rentowności wyznaczonego dwiema metodami

The gas sale revenue  $RS$  will be the same regardless of the formula employed to determine that revenue, that is (Czopek, Trzaskuś-Żak 2003; Trzaskuś-Żak 2005):

$$F + V \cdot X = \bar{p} \cdot X \quad (26)$$

In order to explain the above-mentioned doubts expressed by formulas (23), (24) and (25), the authors used an example of one tariff group – W-4, for a selected gas distributing company “X”.

Three possible variants have been analysed simultaneously, including negative profitability as in case of company “X”, and two cases of profitable sale. At the same time, the researchers have assumed possibility to determine costs incurred for supplying gas consumers in tariff group W-4, with distinguished fixed and variable costs.

The analysis covered results of company “X” in three successive years, while the number of gas consumers – 7000 has not been changing throughout the analysed period, same as charges and tariff prices in group W-4, that is:

- the sum of variable revenue parameters:  $V = 0.8274 \text{ PLN/m}^3$ ,
- the sum of fixed revenue parameters:  $F = 70.10 \text{ PLN/month/1 buyer (consumer)}$ , that is  $70.10 \cdot 12 = 841.20 \text{ PLN/year/1 buyer (consumer)}$ .

The structure of costs has not changed as well, that is:

- fixed costs:  $C_f = 74\,000.00 \text{ thousand PLN/year}$ ,
- variable unit cost:  $c_v = 0.30 \text{ PLN/m}^3$ .

Calculation results are given in Table 1.

TABLE 1

Results for Distributing Company "X", for W-4 tariff group in the analysed year

TABELA 1

Wyniki dla Spółki Dystrybucyjnej „X”, dla grupy taryfowej W-4 w analizowanym roku

Item	Specification	Calculation results		
		7000	7000	7000
1.	Number of buyers/consumers	7000	7000	7000
2.	Gas sale volume, thousand m <sup>3</sup>	120 000	130 000	140 000
	Sale revenue according to W-4 tariff, thousand PLN, including:	105 176.4	113 450.4	121 724.4
3.	3.1. fixed component of revenue, thousand PLN	5 888.4	5 888.4	5 888.4
	3.2. variable component of revenue, thousand PLN	99 288.0	107 562.0	115 836.0
	3.4. variable price component, thousand PLN	0.8274	0.8274	0.8274
4.	Average gas sale price, PLN/m <sup>3</sup>	0.87647	0.872695	0.86946
	Operating costs, thousand PLN, including:	110 000.0	113 000.0	116 000.0
5.	5.1. fixed costs, thousand PLN	74 000.0	74 000.0	74 000.0
	5.2. variable costs, thousand PLN	36 000.0	39 000.0	42 000.0
	5.3. variable unit cost, PLN/m <sup>3</sup>	0.30	0.30	0.3
6.	Operating profit, thousand PLN	-4 823.60	450.40	5 724.4
7.	Break-even point according to method used in practice, according to formula (22), thousand m <sup>3</sup>	128 367.478	129 213.63	129 947.67
8.	Break-even point according to the developed method, according to formula (13), thousand m <sup>3</sup>	129 145.999	129 145.999	129 145.999

### Summary

Obtained results provide grounds to present some general conclusions.

- Break-even point value computed using method employed in practice by distributing companies (formula 22) changes with changing sold gas volume, and more precisely, increases with growing gas sale volume.
- Break-even point value computed using method proposed in this article (formula 13) remains the same regardless of sold gas volume.
- Break-even point value computed using method employed by distributing companies (formula 22) is:
  - higher than break-even point calculated using the proposed method (formula 13) in case of profitable company activity,
  - lower than break-even point calculated using the proposed method (formula 13) in case of unprofitable company activity.

It means that the use of current method for computing break-even point gives an illusion of lower safety margin –  $M_{x1}$  than it is in reality –  $M_x$ , which is illustrated by the example in Fig. 3. The scope of profitable activity is higher than it would arise from the  $BEP_1$  value. Increase in break-even point value accompanying increasing volume of sold gas, computed using the method of distributing companies, results from the fact that when sale  $X$  rises, average sale price drops (formula 10), and then denominator value in formula (22) drops as well.

Appropriateness of formula (13) application to determine gas sale break-even point is proven by the following facts:

- break-even point value cannot change with changing sold gas volume, which is observed when using formula (22),
- the value of revenue calculated using formula (3) is lower than **revenue** computed according to equation (9), these values become equal only when sale  $X$  level is reached, while revenue value has to be the same regardless of employed formula.

Obtained results confirm previously specified conclusions, which may be generalised as follows.

If distributing company sells less gas than quantitative break-even point, then application of the method used so far gives lower value of quantitative break-even point compared to the proposed method. As a result of this, unprofitable operation range and quantitative safety margin are lowered.

If Distributing Company sells more gas than break-even point (Fig. 3), that is it reaches positive profitability, break-even point determined using conventional method has higher value than that calculated using the proposed method.

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**PRZYPADK DWUSKŁADNIKOWEGO PRZYCHODU PRZY WYZNACZANIU WIELKOŚCI GRANICZNYCH  
W METODZIE CVP (COSTS-VOLUME-PROFIT ANALYSIS)**

Słowa kluczowe

Metoda CVP, próg rentowności, wielkości graniczne, cena

Streszczenie

Artykuł przedstawia analizę nowego podejścia do wyznaczania wielkości granicznych, to znaczy: granicznej wielkości sprzedaży, granicznej wartości sprzedaży, granicznej ceny, granicznych kosztów stałych i granicznego kosztu jednostkowego zmiennego, przy uwzględnieniu przychodu dwuskładnikowego. Stały i zmienny składnik przychodu wynika ze specyfiki w naliczaniu ceny przez przedsiębiorstwa dystrybucji gazu, która składa się ze stałej części (abonamentowej) i części zmiennej zależnej od wielkości sprzedaży. Ta szczególna sytuacja ma miejsce w sektorze energetycznym i telekomunikacyjnym, czyli dystrybucji gazu i energii elektrycznej, jak również w dystrybucji usług telekomunikacyjnych.

Uwzględniając specyfikę wymienionych sektorów gospodarki można stosować zaproponowaną w niniejszym artykule metodę do wyznaczania wielkości granicznych.

W przypadku dystrybucji gazu, o wielkości przychodu w znaczący sposób decyduje obowiązująca taryfa. Opracowaną metodę wykorzystać można przy tworzeniu taryf sprzedaży gazu w przedsiębiorstwie gazowniczym. Jest to istotne ze względu na fakt, że od konstrukcji wewnętrznej taryfy zależy przychód całkowity, ale również jego składowe, czyli przychód stały i zmienny. Stosując zaproponowaną metodę można ocenić sytuację przedsiębiorstwa, wyznaczyć wielkości graniczne, marginesy bezpieczeństwa, a także koszty i przychody (stałe i zmienne), wyodrębnić słabe punkty jego funkcjonowania, zbadać ich przyczyny, a także kierunki i sposoby poprawy istniejącej sytuacji. Próg rentowności liczony metodą stosowaną w praktyce spółek dystrybucyjnych zmienia się wraz ze wzrostem sprzedaży gazu, natomiast przedstawiona nowa metoda pozwala uniknąć tej sytuacji i daje wynik bardziej precyzyjny. Oznacza to, że stosowanie dotychczasowej metody obliczania progu rentowności daje złudzenie mniejszego marginesu bezpieczeństwa niż to ma miejsce w rzeczywistości, biorąc pod uwagę rentowną działalność. W przypadku nierentownej działalności spółki, próg rentowności liczony według dotychczasowej metody jest mniejszy od wyznaczonego według proponowanej metody, co skutkuje zaniżeniem nierentownej działalności. Reasumując można stwierdzić, że wyznaczanie wartości wielkości granicznych za pomocą proponowanej metody pozwala na dobór bardziej precyzyjnej strategii zarządzania cenami i kosztami w przedsiębiorstwach gazowniczych.

**THE CASE OF TWO-COMPONENT REVENUE WHILE DETERMINING BOUNDARY VALUES  
IN THE COSTS-VOLUME-PROFIT (CVP) ANALYSIS**

Key words

CVP method, sale revenue, break-even point, boundary values, price

Abstract

The article presents an analysis of a new approach to determining boundary values, that is: boundary sale volume, boundary sale value, boundary sale price, boundary fixed costs and boundary variable unit cost, taking into account two-component sale revenue. Fixed and variable component of revenue results from specificity in price calculation by gas distribution enterprises. The price consists of fixed part (standing charges) and variable part dependent on sale volume. This particular situation is observed in power and telecommunication industries, that is distribution of gas and power and delivery of telecommunication services.

Taking into account the specificity of sectors listed above it is possible to use the proposed method for estimation of a boundary values.

In the gas distribution business the value of revenue is dependent on valid gas tariffs. The new developed method for construction of gas tariffs. This is very important issue because the construction of tariffs depends on whole revenue, that is mean, fixed and variable parts of revenue.

According to this new method, it is possible to estimate the situation of a gas company, economic marginal volumes, marginal safety factors, costs and sale revenue (fixed and variable parts of them), mark out the weak points of gas companies activity, point out their reasons and the ways or directions of improving current situation. The break-even point, determined by method used in practice by gas distribution companies, is changing (growing) with a volume of gas sale while the new method lets avoid this situation and gives more precise result. It means that using previous method for calculation of a break-even point gives the illusion of lower value of safety margin, than it is in fact taking into account profitable enterprise activity. In the case of unprofitable company activity the break-even point calculated using current method is lower than this computed by new method resulting in an underestimation of unprofitable enterprise activity.

Summarizing, the designation of marginal volumes by using proposed method lead to more precise choices of management of prices and costs in gas companies.